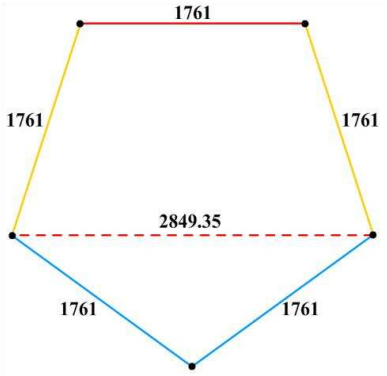






A New Canon of 'Divine Proportion' in Leonardo da Vinci's 'Last Supper'

CAN WE FINALLY DISPENSE WITH BRACHERT'S MUSICAL CANON?

Carl Balowski | February, 2024

Abstract

This study critically examines Thomas Brachert's hypothesis that Leonardo da Vinci's "The Last Supper" is structured according to musical proportions, specifically a Pythagorean 12:6:4:3 ratio. Brachert's theory, linking the painting's spatial organization to mathematical ratios from music theory, posits that Leonardo integrated elements of musical harmony into its design. However, discrepancies highlighted by recent advanced technological analyses, such as failures in Brachert's grid to align with the painting's actual dimensions and the supposed harmonic tapestry alignments, call into question the accuracy of this musical proportion theory.

Further analysis reveals a significant alignment within the painting that more closely adheres to the principles of Pacioli's "Divine Proportion" (Golden Ratio) rather than the musical ratios proposed by Brachert. This finding, reflecting the geometric and arithmetic precision Leonardo likely applied, informed by his collaboration with Luca Pacioli, suggests an intentional use of a more complex canon of proportion that extends beyond mere musical harmony. This challenges the previously accepted musical proportion theory, proposing instead that Leonardo aimed to encode a deeper, volumetric unity within the painting, potentially reflecting the Euclidean construction of the dodecahedron as articulated in Pacioli's Da Divina Proportione.

The study raises questions about the adequacy of the musical proportion theory and suggests that Leonardo might have favored a system of proportion influenced by his academic and professional collaboration with Pacioli. Forthcoming research will aim to delve deeper into these issues, exploring the potential integration of a previously unidentified canon of proportion in "The Last Supper" that aligns more closely with Leonardo's interdisciplinary approach to art, science, and mathematics. This current study focuses more upon the inadequacy of Brachert's basic hypothesis and posits that "The Last Supper" deliberately embeds complex geometric and arithmetic principles, challenging the previously accepted 'canon of musical proportions' and inviting further investigation into the Divine Proportion's role within Leonardo's masterpiece.

Introduction

In the 21st century, armed with advanced tools and techniques, we can now conclusively demonstrate that Brachert's widely accepted 'Musical Canon of Proportion' in Leonardo's Last Supper is fundamentally flawed. [1] It is imperative that we replace it with a canon that comprehensively encompasses the full spectrum of Leonardo's interests during the last decade of the 15th century. While this revision may raise concerns among Leonardo scholars and fellow enthusiasts of 'Sacred Geometry,' it is crucial to recognize that Brachert's work has enjoyed a long history of citation by respected scholars dedicated to the study of Leonardo.[2] However, I'll demonstrate in my next study, that as he painted the Last Supper, Leonardo likely employed a more precise and contextually relevant canon of proportion, as illustrated in "The Divine Proportion," a collaborative effort with his mathematical mentor, Luca Pacioli.

Sight vs Hearing

Within Leonardo's notebooks, we encounter analogies illustrating how musical proportions blend harmoniously, much like a painter employing colors, lines, and perspective to craft a visually harmonious and aesthetically pleasing painting. However, it's crucial to recognize that both Leonardo and Luca Pacioli considered 'hearing' as an inferior sense compared to 'seeing,' which serves as an architecturally symbolic "door to the intellect." Since music is rooted in the sense of hearing, it should not be regarded as a true mathematical science on the same level as Arithmetic, Geometry, and Astronomy, especially if Perspective, founded on the superior sense of sight, is not also acknowledged as a legitimate mathematical science.

Pacioli proposed that there should not be merely four scientific disciplines but either three or five. If Music were to be considered the fourth discipline, then Perspective, which he deemed more deserving, should be recognized as the fifth discipline. Alternatively, if one hesitates to acknowledge Perspective as a scientific discipline, then Music should not be acknowledged

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[1] Brachert, T. (1971). A Musical Canon of Proportion in Leonardo da Vinci's Last Supper. *The Art Bulletin*, *53*(4), 461–466. Brachert supposed that Leonardo created the Last Supper using a 6:12 grid and that he positioned key elements in accord with constant musical proportions, where a ratio of 1:2 (6:12) is the musical *octave*, a ratio of 2:3 (8:12) is the musical *fifth* and a ratio of 3:4 (9:12) is the musical *fourth*.

[2] See Matthew Landrus, Rational Estimates for Irrational Problems: Proportional Geometry in the Work of Leonardo da Vinci; Kemp (Marvellous Works …).

either, for Perspective's superiority over Music is evident, as it draws upon both the superior sense of sight and intellectual reasoning, serving as the "door to the intellect." Pacioli grappled with the conflict between his personal conviction regarding Perspective's superiority over Music as a scientific discipline and the authority of scientific tradition that suggested otherwise. This 'moral dilemma' weighed heavily on his spirit for several years. His ultimate proof in favor of Perspective's superiority over Music stemmed from his personal sensory experience of Leonardo's Last Supper, describing his experience of it in full.[3] According to Pacioli, Leonardo's use of perspective works its magic by elevating the perception of the gracefully composed figures to a realm that is nothing short of divine. The artist vividly captures the apostles' immediate and palpable reactions to the voice of the infallible truth when it pronounces, "One of you will betray me", leaving an indelible mark on the observer's soul.

Pacioli is unequivocal in his belief that the arithmetical and geometrical proportions of Perspective surpass the harmonic proportions of Music as a scientific discipline. Even in his own notes, when musing about the scientific disciplines and the superiority of the eye, Leonardo echoes Pacioli's assessment of Perspective's superiority over Music.[4] Of course, Pacioli's and Leonardo's valuation of Perspective over Music doesn't negate their use of musical proportions

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[3] Tennenbaum, Luca Pacioli's 1498 'On the Divine Proportion', p. 9. "*If these scholars say that music satisfies the hearing, one of the senses given us by nature, I say that perspective satisfies the sight, which is much more worthy, as it is the first door of the intellect; if they say that music applies to the sonorous number, and to the implied measure of its extension in time, then I say for my part that perspective observes the natural numbers according to all its definitions, such as the measure of a visual line. If the former is recreation for the soul through harmony, then the latter does this by the necessary distance and the very delectable variety of its many colors. If the former respects its harmonic **proportions**, then the latter, respects the arithmetic and geometric **proportions**. In brief, excellent Duke, for many years and until now, this question has tormented my spirit; nothing makes clear to me why four should be better than three or five, even if we say that such a consensus of scholars could not err. Still, whatever they may say, it does not remedy my ignorance, **for who is it that sees a graceful figure with its necessary lineaments well placed,** where the only thing missing seems to be the breath, does not judge it a thing divine rather than human? Painting imitates nature as much as can be done and this appears clearly to our eyes when we look at that marvelous representation of the ardent desire of our salvation.[3] In that painting it is not possible to imagine the apostles more alive to the sound of the voice of the infallible truth when he says 'One of you will betray me'; where with acts and gestures to one another, with living and painful astonishment, it appears that they are speaking! So justly did Leonardo depict this moment for us with his skillful hand."*

[4] Kemp, Leonardo On Painting, p. 14-15 Leonardo writes: "The sciences termed mathematical are those that, passing through the senses, are certain to the highest degree, and these are only two in number. The first is arithmetic, and the second is geometry, one dealing with discontinuous quantity and the other with continuous quantity. From these two sciences, perspective is born, devoted to all the functions of the eye and its delight with various speculations. These three disciplines—arithmetic, geometry, and perspective—are indispensable, and the absence of any one of them hinders accomplishment. Next in line is music, which arises from both continuous and discrete quantities and is dedicated to the ear, a sense less noble than the eye."

and its language in their various works. However, if a proportional system that better represented Perspective -- by emulating architectural design -- were available to Leonardo, it stands to reason that he would apply it, over Music, as the symbolic 'door to the intellect' in his painted architecture. With this in mind, we must first take a critical look at Thomas Brachert's in-depth analysis of Leonardo's 'Last Supper,' which introduced a compelling hypothesis that Leonardo meticulously designed the painting in accordance with musical proportions.

Brachert's Canon

Brachert, exploring the sophisticated interplay between the realms of art and mathematics—a domain where Leonardo's profound interests notably intersected—suggests that Leonardo may have intentionally woven elements of musical harmony into the painting's spatial organization. According to Brachert, Leonardo could have harnessed the mathematical ratios prevalent in music theory to thoughtfully arrange the components of his painting, thereby achieving a visual harmony that mirrors the structured proportions inherent in musical composition.

This hypothesis is deeply anchored in the ancient philosophies of Pythagorean tuning and the "music of the spheres," a concept that seeks to elucidate the proportional relationships and movements within nature and the cosmos as akin to musical harmony. As mentioned, Leonardo was a polymath whose curiosity and expertise spanned a broad spectrum of disciplines, so it is conjectured that he may have applied these harmonious principles to his artistic endeavors. Brachert's examination suggests that various elements depicted in "The Last Supper"—ranging from the architectural design of the depicted room, the deliberate placement of the apostles, to the overall geometric configuration of the painting—may correspond to specific musical intervals or chords.

Advocates of Brachert's theory rightly draw attention to Leonardo's documented fascination with the concepts of proportion, geometry, and the synthesis of diverse realms of knowledge, including music. They argue that such a comprehensive, multidisciplinary methodology could align seamlessly with Leonardo's established practices e.g., Albertian construzione legittima and reticolato. Building on this foundation, Brachert speculates that Leonardo's architectural rendering of the biblical 'upper room' adheres to a meticulously defined 12 x 6 grid, symbolizing a musical octave, which Leonardo aimed to divide into precise thirds, halves, and quarters

(Figures 1 and 2).[5] Through Brachert's lens, the rear wall of the depicted upper room spans four grid blocks in width, with each of the room's side walls, portrayed in perspective, also extending four grid blocks in width. This division segments the twelve horizontal blocks of the painting into three equal portions, with the entire width of the painting, measured at 8805 mm, being equally divided into segments of 2933.33 mm each—thereby totaling the full width when multiplied by three. IN this way Brachert's hypothesis meticulously posits that "The Last Supper" is divided into precise mathematical proportions, aligning with a Pythagorean 12:6:4:3 musical ratio, suggesting a deliberate orchestration of the painting's elements to create an optical illusion that the design is rooted in musical harmony.

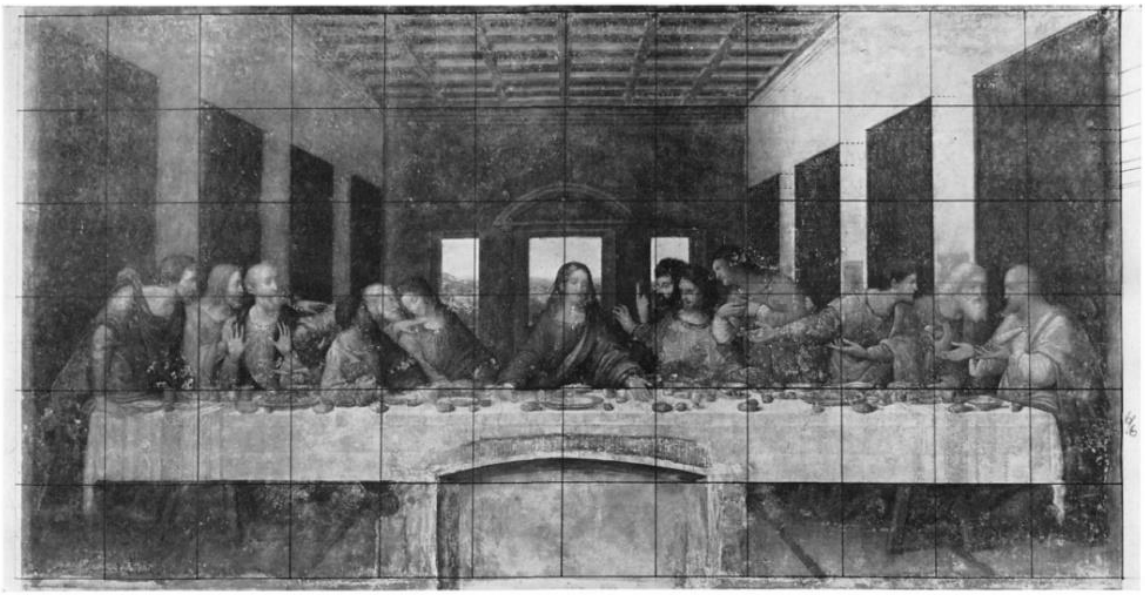


Figure 1

[5] As matter of practice and convenience there is little doubt that Leonardo used an Albertian reticolo (pictorial grid) in order to transfer his small scale sketches to a large scale wall, however this does not mean that the proportional theme that Leonardo wished to convey had to align thematically with sectors of the grid. Like the paintbrush, the grid was a tool.

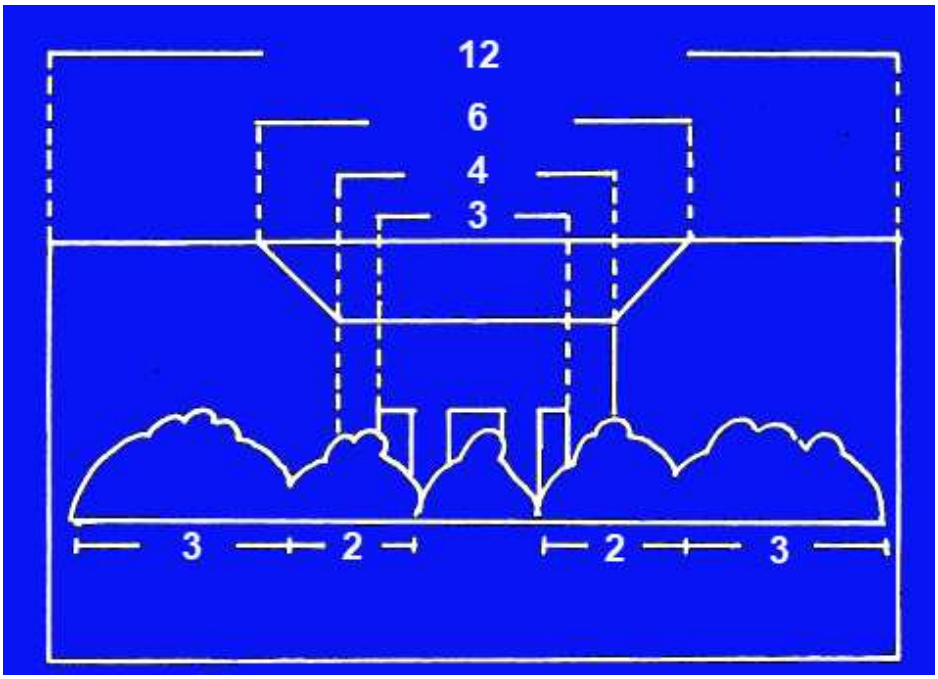


Figure 2

At the outset, Brachert's analysis presents a well-articulated argument, suggesting a hidden layer of mathematical and musical structuring woven into the fabric of Leonardo's masterpiece. However, a measured exploration into the specifics of Brachert's analysis uncovers the inherent complexities and potential challenges of such an interpretation. The hypothesis of a seamless integration between mathematical, musical, and artistic principles necessitates rigorous scrutiny. Modern advancements, notably the ultra-high-resolution digital imaging and plotting capabilities offered by Haltdefizione, now enable precise measurement of every aspect of the painting. These include its overall dimensions, the width of the painted upper room's rear wall, the length and height of the table, dimensions of doors and windows, sizes of ceiling coffers, tapestries, and the positioning of figures, among others.

For analytical clarity, it's essential to acknowledge Leonardo's foundational understanding of the painting's wall as 8805 millimeters, equating to exactly 15 braccia, a measurement corroborated both by Haltdefizione and by line AB in Pacioli's "Da Divina Proportione." This same unit of measurement, the braccio (587 mm), finds relevance in the architecture of Santa Maria delle

Grazie in Milan and the Old Sacristy and Pazzi Chapel in Florence, further embedding Leonardo's work within a broader architectural and mathematical context.[6]

Brachert's superimposed 12:6:4:3 grid, however, seems to stretch beyond what might be considered an acceptable margin of human error. He proposed that the upper room's painted rear wall precisely constitutes one-third of the painting's width. This assumption introduces significant inaccuracies, especially considering Brachert's decision to use the inside bottom (rather than the outside bottom) of the top architrave as the painting's 'upper line' and setting the baseline of the painting nearly 20mm above a distinct paint line left by Leonardo. Such choices appear to reduce the total height of the painting by several centimeters so that his 12 x 6 rectangular grid would seemingly conform exactly to Leonardo's painting. Despite this manipulation, Brachert's grid still overshoots the painting's total width by about 3 centimeters on the left side and misaligns with the rear-wall by 4.28 centimeters on each side, totaling a significant discrepancy of 8.56 centimeters. This pivotal flaw in Brachert's analysis stems from the now-questioned assumption that Leonardo intended for the width of the upper room's painted rear-wall to precisely be one-third the width of the entire painting, with the tapestries and table edges positioned in 'harmony' with this division.

When scrutinizing Brachert's conjectured rear-wall measurement of 2935 mm (purportedly one-third of the painting's width) against the actual rear-wall measurement of 2849.35 mm, a deviation of 85.64 millimeters, or 2.9%, is revealed. This variance significantly impacts the presumed proportional system Brachert suggested, for if his grid is adjusted to align with the exact width of the upper room's rear wall (2849.35 mm), the remaining vertical grid lines fail to match the so-called 'proportional' borders of the tapestries or the painting's overall width (Figures 3 and 4). The discrepancy is also starkly evident when enlarging his grid to fit the painting's exact width (Figure 5).

[6] Carl Balowski, 'Analysis of Leonardo da Vinci's Braccio at Santa Maria delle Grazie: The Foundation of a Medici Inspired Convent in Milan'

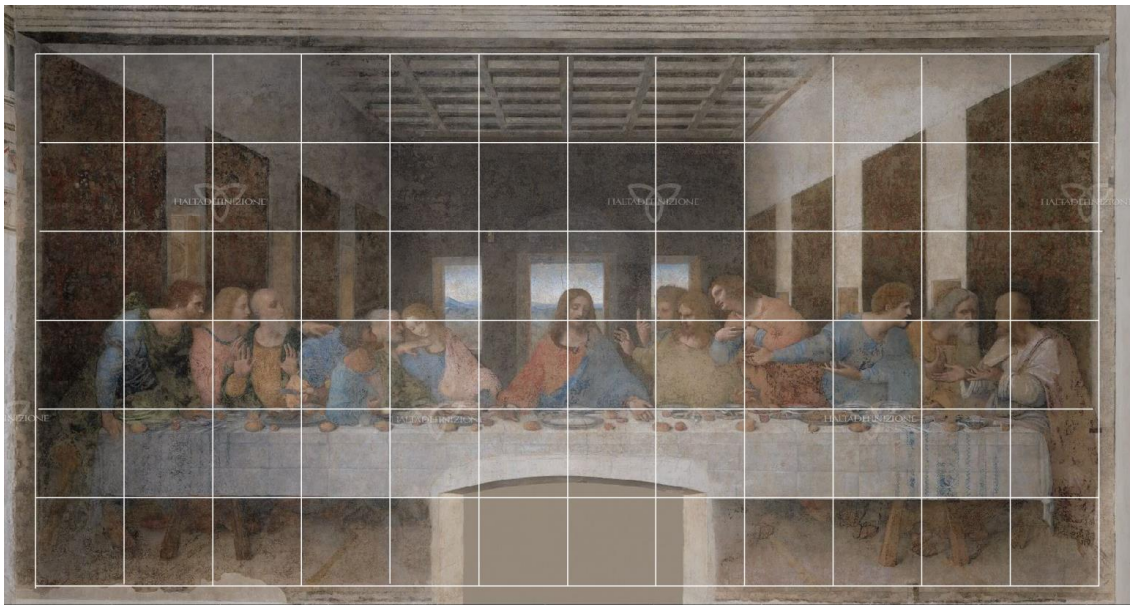


Figure 3

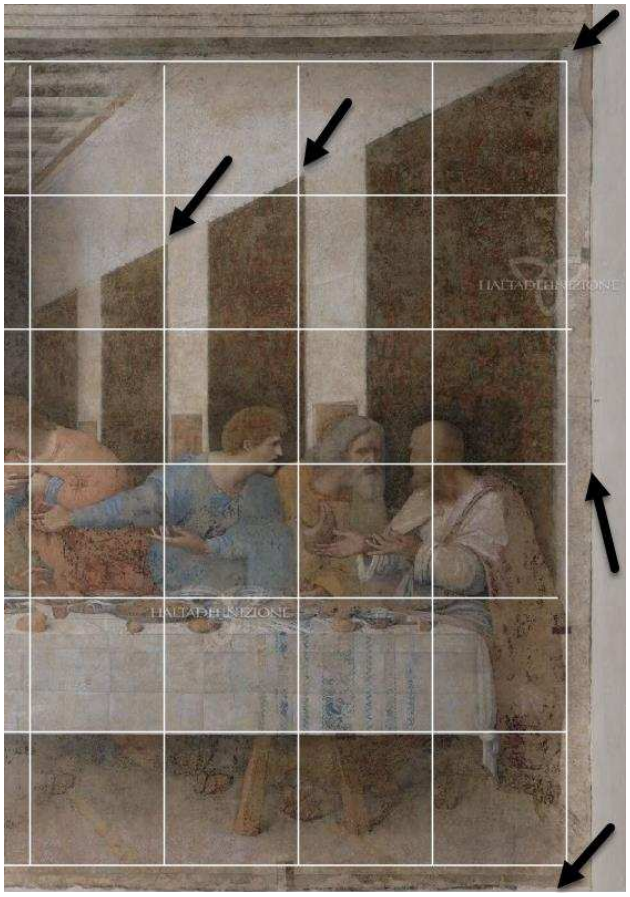


Figure 4

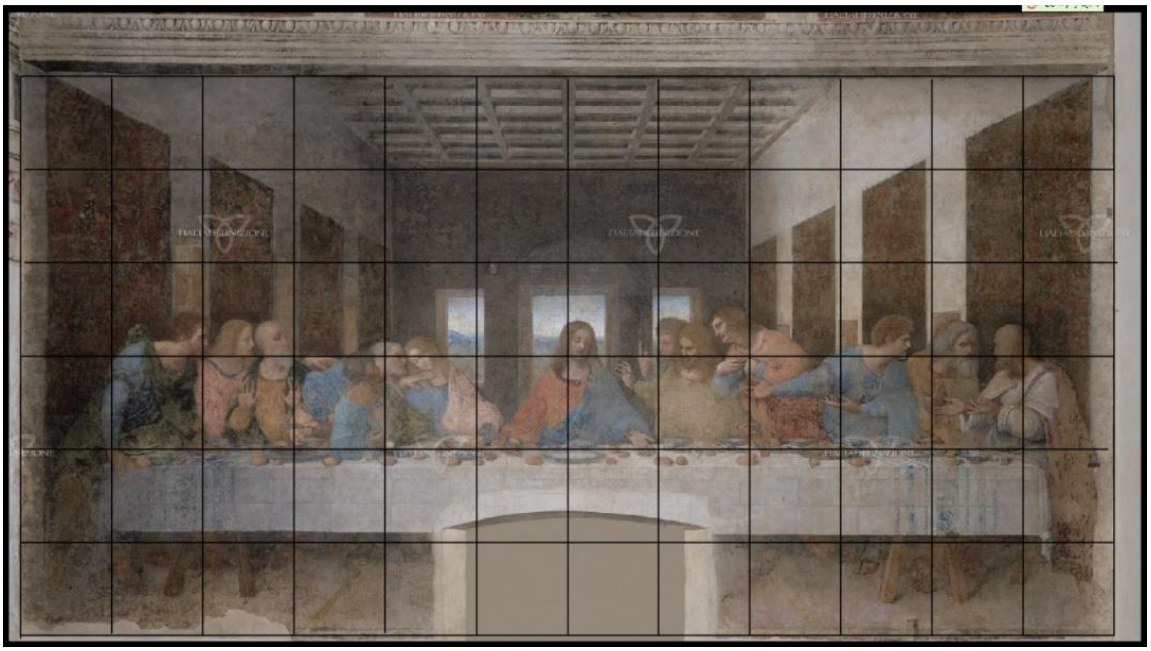


Figure 5

Delving deeper into Brachert's methodology reveals that when the rear wall's total length of 2849.35 mm is divided by four, according to his proposed grid, it suggests that each grid segment should measure precisely 712.33 mm. This calculation is based on the division of 2849.35 mm by 4. Adhering to Brachert's theory, if we assume that the nearest edge of the second-largest tapestry is positioned two grid segments from the rear wall, and the nearest edge of the third-largest tapestry is one grid segment from the rear wall, then theoretically, the distances to these tapestry edges should measure 1424.6 mm and 712.3 mm, respectively. However, the actual measurements recorded diverge from these theoretical values, being 1450.9 mm and 725.5 mm instead. These figures reflect an error margin of 1.8% relative to the rear wall—which itself deviates by 2.9%—but intriguingly maintain a 2:1 ratio to each other (a subject explored in my next study).

It's clear that Brachert’s grid does not extend to encompass the full height and width of the painting. Furthermore, the tapestries on the side wall, presumed to be in 'harmonic' alignment with the grid, fail to match accurately in any of Brachert’s theoretical configurations. What Brachert perceived as a deliberate design appears, under the meticulous examination of modern restoration techniques and ultra high-resolution measurements, to be more of a dissonance

than a factual representation of musical harmony. It's like playing a C-F# interval, instead of a C-G. However, dismissing Brachert's analysis as entirely flawed would be premature.

Another Proportion

Long ago, when I superimposed and adjusted Brachert's grid to fit Leonardo's painted 'rear wall', a striking observation emerged: the horizontal width of the rear wall almost perfectly matches the vertical distance from the bottom of the tablecloth to the top of the rear wall, with a minor discrepancy of only 6.6 millimeters, or 0.23% (Figure 6). Despite the constraints of perspective, which theoretically would not permit such a symmetry, this near-equivalence of horizontal and vertical measure creates the visual impression that the furthest element in the scene (the rear wall) and the nearest element (the front face of the table) are unified, suggesting the formation of a three-dimensional square, or in essence, a cube.

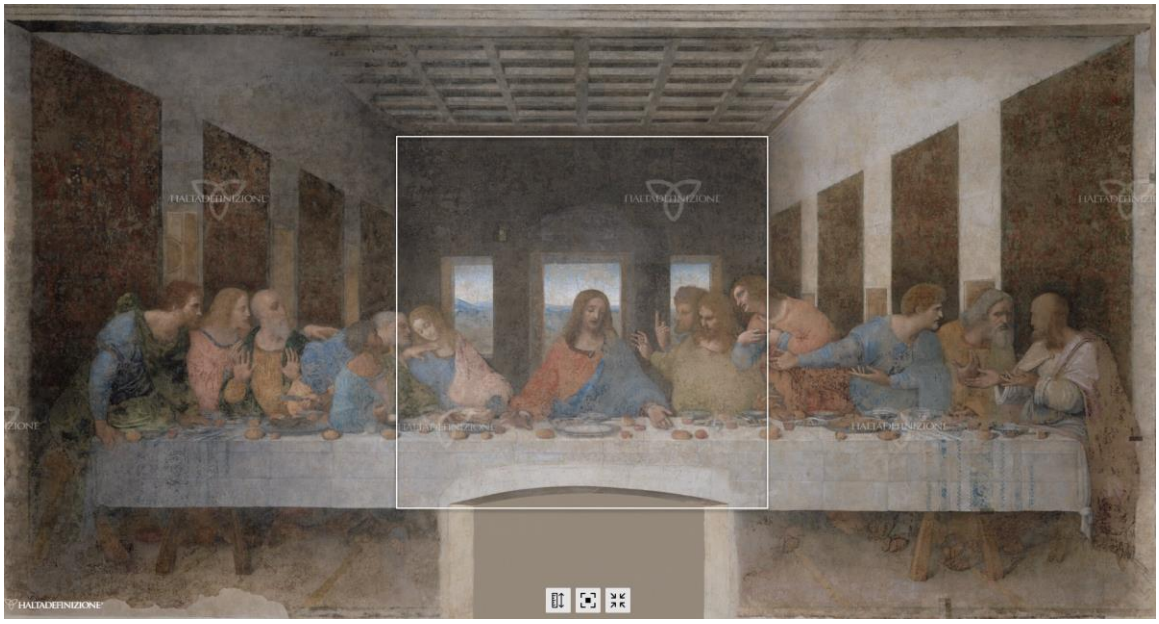


Figure 6

This perceived geometric accord between the foreground and background elements of the painting does not strike me as coincidental. This subtle yet profound equilibrium lends itself to the interpretation that, beyond its immediate visual narrative, the painting may encode proportions related to the cube, intentionally embedded by the artist. The bottom edge of the tablecloth, being the nearest visual element in the room, appears to perfectly 'square-off'

against the rear wall, the farthest element in the room. Extending the edges of the rear wall downwards to meet the bottom of the tablecloth reveals a precise alignment, with two of the painted orthogonal lines on the floor appearing to terminate from the tablecloth's edge to the painting's baseline. This orthogonal effect is mirrored near the painting's topline by the orthogonals of the six-by-six, squared perspective coffered ceiling (Figure 7). These alignments seem far too impressionable to be mere happenstance, suggesting instead a calculated intention by Leonardo to impart a cube-like perception to the room's sacred interior.

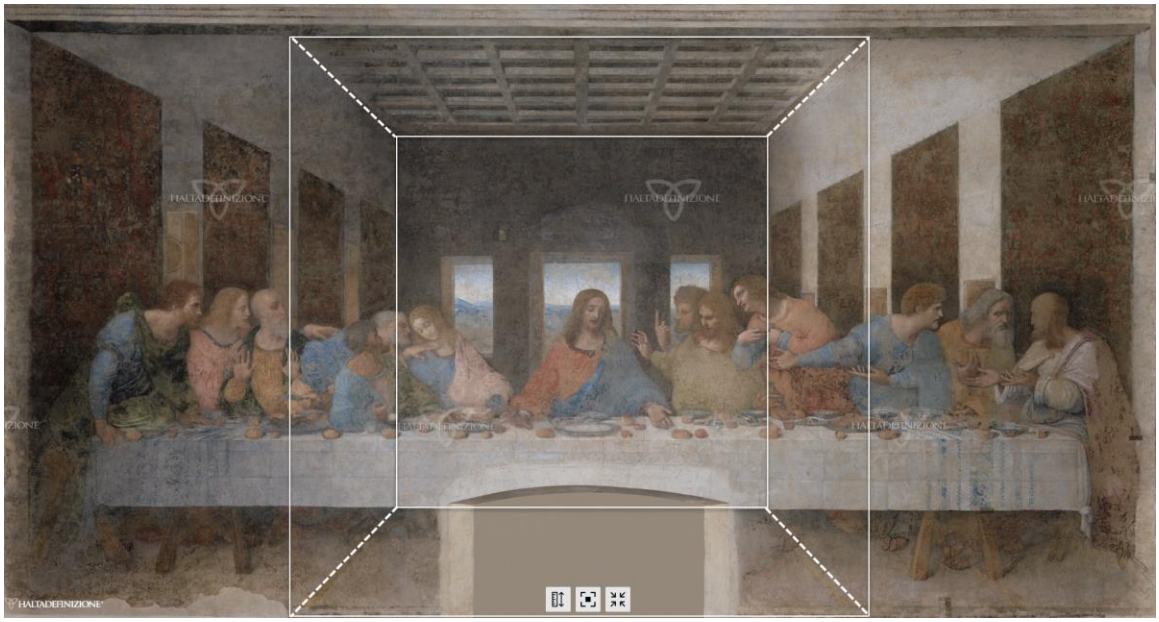


Figure 7

This cubic illusion not only heightens the sanctity of the depicted space but also reimagines the rear wall as more than a mere boundary. It becomes a fundamental component in harmony with the spatial arrangement of the room. This effect draws a profound parallel not only with the architectural principles underlying sacred biblical constructs, notably the Holy of Holies and the New Jerusalem, where the cube symbolizes divine perfection and the intersection of the earthly with the heavenly, but more importantly with Pacioli's (Euclid's) construction of the dodecahedron, the polyhedron with 12 pentagons built upon a cube, also known as the quintessence (Figure 8). This emphasizes how the cube serves as a cornerstone not just in Pacioli's mathematical endeavors but also in the conceptualization of sacred spaces within the

biblical narrative, underscoring a universal reverence for its form as a bridge between the mortal and the divine.

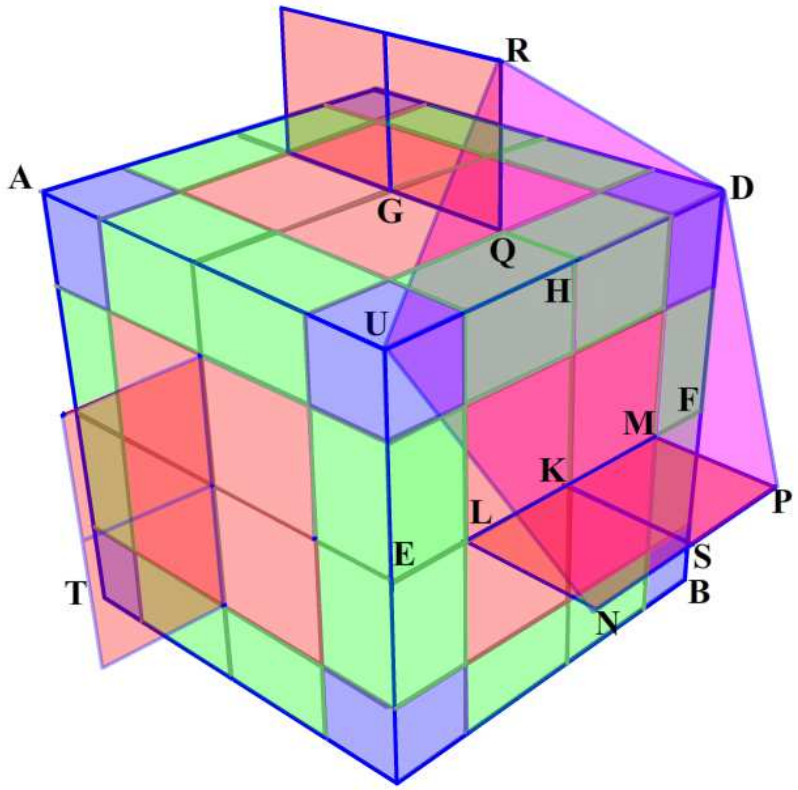


Figure 8

Upon closer examination, the precise ratio between the width of the room's painted rear wall (an illusory extension of the refectory) and the actual width of the refectory wall onto which the painting is rendered, is found to be 3.09:1. This ratio, especially in light of Leonardo's collaboration with Luca Pacioli on "The Divine Proportion," is notably more accurate than Brachert's proposed 3:1 ratio, which deviates by more than 2.9%.

To entertain this analysis, one must first assume that Leonardo had intent for painting the rear wall proportionate to the painting's total width. The painting's width varies from 8794 mm near the baseline to 8810 mm at the topline, therefore we'll take 8805 mm for this analysis. The painted rear wall of the upper room measures 2849.35 mm across, showing minimal variance. Dividing the painting's total width by the width of the painted rear wall yields a ratio of 3.0901:1, signifying that the width of the painting is slightly more than three times the width of

the painted rear wall. To align with Brachert's theory—asserting the painting's width is exactly three times the width of the painted rear wall—the painting would need to be significantly narrower at 8548 mm, not 8805 mm. This calculation reveals Brachert's analysis to be off by more than 257 mm. Even adopting the painting's narrowest width for calculation, Brachert's theory still misses by over 246 mm!

3.0901 is exactly half of 6.1802, a figure that, when considered alongside the number 10, mirrors the Golden Ratio and is exemplified in Euclid 13.5. This relationship, mathematically expressed as √(125) - 5 and (√(125) - 5)/2, aligns with the arithmetic that Pacioli employed in his seminal work "On the Divine Proportion," particularly in his exploration of the underpinnings of the dodecahedron as presented in Euclid's "Elements." It is within this context of interdisciplinary synthesis that Leonardo, while concurrently working on "The Last Supper," provided the illustrations for Pacioli's treatise, immersing himself in the study of mathematical harmony and geometric precision while meticulously 'drawing' the Platonic solids.

This convergence of artistic endeavor and mathematical exploration raises significant questions about Leonardo's intentions. Was the proportional alignment of the rear wall's width with the overall width of the painting a mere coincidence, or did it reflect Leonardo's deliberate application of principles gleaned from his collaboration with Pacioli? Was Leonardo attempting to manifest the Divine Proportion, a concept he encountered through his study of arithmetic and Euclidean geometry under Pacioli's guidance, within the spatial dimensions of "The Last Supper"?

Moreover, this analysis challenges the adequacy of Brachert's Canon of Musical Proportion, suggesting the possibility that Leonardo might have adhered to a different, perhaps a more sophisticated and exacting, canon of proportion, deeply rooted in the mathematical and geometric insights that informed a religious perspective of cosmic and artistic harmony.

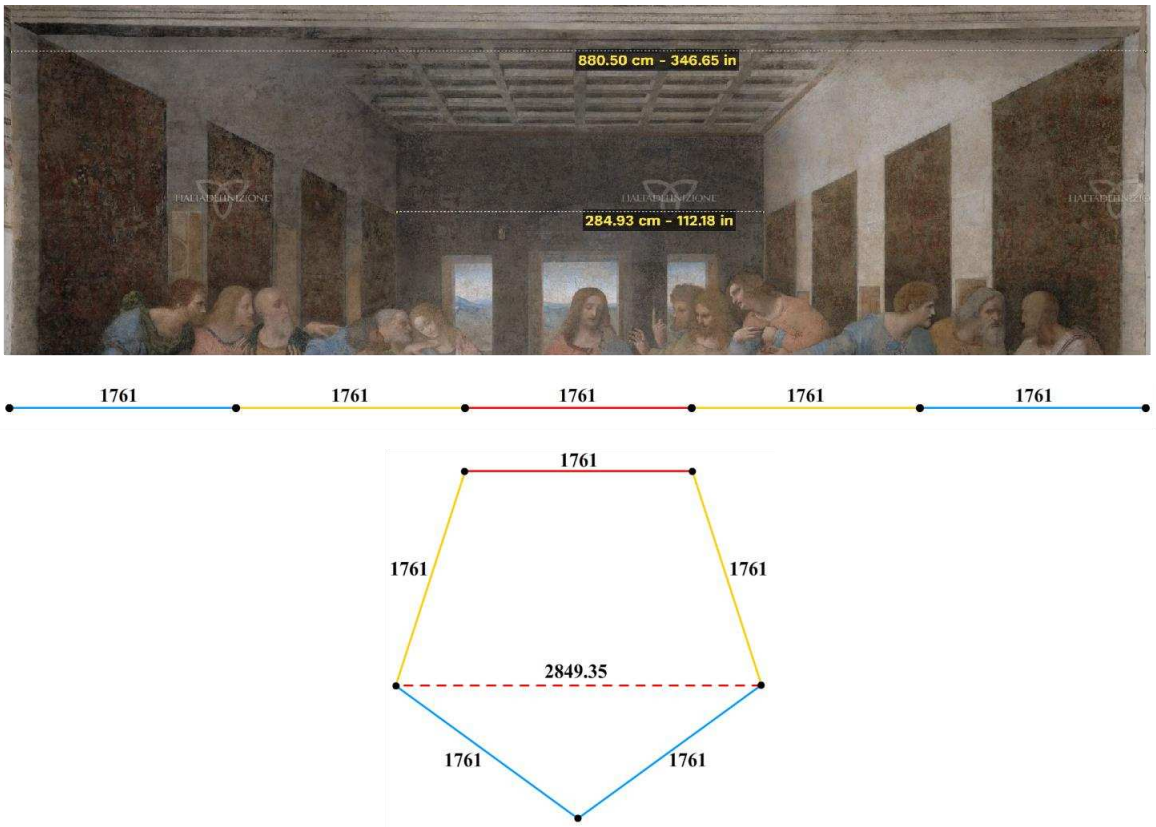


Figure 9

These reflections set the stage for an in-depth analysis of Leonardo da Vinci's application of mathematical precision and proportional reasoning in "The Last Supper." To this end, a demonstration that Leonardo integrated a previously unidentified canon of proportion based on Pacioli's Da Divina Proportione into "The Last Supper" is the central theme of my next work. In the interim, I invite you to contemplate the fascinating 'coincidence' that a pentagon, each side measuring three braccia (the measure of man),[7] not only boasts a perimeter that precisely

[7] A braccio was 587 mm, therefore three braccia was 1761 mm. The average proportional measure of man was considered to be three braccia, and this height was first used in perspective constructions by Alberti. Leonardo, Notebooks: "If a man 2 braccia high is too small, one of four is too tall, the medium being what is admirable. Between 2 and 4 comes 3, therefore take a man of 3 braccia in height and measure him by the rule I will give you. If you tell me that I may be mistaken, and judge a man to be well proportioned who does not conform to this division, I answer that you must look at many men of 3 braccia, and out of the larger number who are alike in their limbs choose one of those who are most graceful and take your measurements." And Alberti: "Then along the perpendicular lay out three braccia from the bottom line. At this height draw a very fine line. Then make a point either above, below, or exactly on this line, either in the middle or at the side. If you want the things to appear head-on, place your point in the middle of this line." And Pacioli: "Now to find the diminution of the one and the other, you say as follows: If the hypotenuse ob, which

equals 8805 mm but also features diagonals that measure exactly 2849.35 mm—mirroring the ratio of the painting's overall width to that of its painted rear wall (Figure 9). This geometric curiosity which underpins the core of Pacioli's treatise, further entwines Leonardo's artistic endeavors with his profound engagement in the realms of geometry and mathematical harmony, hinting at deliberate manifestations of the Divine Proportion within his Cenacolo.

is √712, gives me a height of 3, which we said is the height of the men, what will the hypotenuse od. which is 10. give me. Multiplying and dividing you will find that it gives you √ 1 47/178. That is how large the person al b will present himself to you"

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