

Analysis of Leonardo's Braccio at Santa Maria delle Grazie

The Foundation of a Medici Inspired Convent in Milan

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Abstract

In this first paper of a two-part series, I present a novel and detailed analysis of the statement made by Luca Pacioli in 'De Divina Proportione' about the specific braccio measurement that Leonardo da Vinci utilized in the creation of the Sforza 'horse' monument. My research extends beyond this to examine how this unit of measure is not only integral to Leonardo's 'Sforza horse' but is also a key element in his renowned work, 'The Last Supper'. Furthermore, I investigate the roots and implementation of this measurement at Santa Maria delle Grazie, offering insights into its historical and artistic relevance within the broader scope of Leonardo's oeuvre.

A key aspect of my study is the architectural and historical connections between the church of San Lorenzo in Florence and Santa Maria delle Grazie (SMG) in Milan. I propose that the 587 mm braccio used in Leonardo's design of 'The Last Supper' at SMG was likely inspired by its use in San Lorenzo. This hypothesis is supported by my findings on the axis alignments and measurement parallels between San Lorenzo and SMG, underscoring the church's importance in Florence's religious heritage and the Medici's role in strengthening the traditional ecclesiastical bond between Florence and Milan.

In my research, I analyze the architectural layout of Santa Maria delle Grazie, particularly focusing on the proportional relationships and symmetries between the refectory, where 'The Last Supper' is located, and other parts of the complex, such as the Cloister of the Dead and the church's tribune/dome. My findings suggest a deliberate and theologically informed design, harmonizing with the convent's existing structures and reflecting similar measurement systems used in San Lorenzo.

In my forthcoming paper, I will demonstrate Leonardo's use of the 587 mm braccio in cooperation with Pacioli's Divine Proportion (the Golden Ratio) in 'The Last Supper'. This will be supported by detailed measurements and comparative studies.

In conclusion, my research presents a comprehensive analysis of the use of specific measurements in Leonardo da Vinci's works, influenced by Luca Pacioli, and their reflection in the architecture of Santa Maria delle Grazie. It sheds light on the intricate connections between art, mathematics, and architecture in the Renaissance, highlighting the profound impact of the 587 mm braccio measurement from its roots in San Lorenzo to its application in Milan, thereby revealing the complex web of architectural, political, and artistic exchange during this period.

Reader's note: Figures 5 – 7 are located at the end of the paper.

Born in San Sepolcro and considering himself a Florentine compatriot of Leonardo, Luca Pacioli, a Franciscan friar, emerged as a popular and influential mathematician in the late 15th century. [1] By early 1495, Leonardo da Vinci had acquired a copy of Pacioli's newly published 'Summa de Arithmetica', a comprehensive book on mathematics written in vernacular Italian. This work covered a range of topics including algebra, Euclidean geometry, some trigonometry, proportions, and accounting. While much of the content in the 'Summa' was not original, its significance lay in the coherent summarization of extensive mathematical knowledge in a single volume, making it an instant success.

This success, possibly bolstered by encouragement from Leonardo and his host, Ludovico Sforza (Sanseverino), led to Pacioli's arrival in Milan in early 1496.[2] In Milan, Pacioli found in Leonardo a keen artist/engineer, eager to deepen his understanding of mathematics. This period saw Leonardo's growing interest in geometry and arithmetic, as evident in his notebooks from that time. Pacioli and Leonardo quickly developed a close friendship and collaborative relationship. During this phase of intense study by Leonardo, Pacioli began writing the 'sequel' to his 'Summa', titled 'De Divina Proportione' (The Divine Proportion). This treatise focused on the Golden ratio and the Platonic/Archimedean solids, and Leonardo meticulously provided 60 illustrations for it.

Interestingly, both Pacioli's 'De Divina Proportione' and Leonardo's 'The Last Supper' were completed around the same time at the end of 1498. Pacioli's work is notably the first written account acknowledging the completion of Leonardo's masterpiece. This suggests that for nearly three years, from early 1496 to late 1498, while Pacioli worked on 'De Divina Proportione' and taught mathematics to Leonardo, Leonardo was simultaneously working on 'The Last Supper', studying mathematics, and creating illustrations for Pacioli's treatise.

The mutual influence between Leonardo and Pacioli was significant. Pacioli entrusted Leonardo with the illustration of his important book on the Golden Ratio and polyhedra, and he also expressed extensive praise for Leonardo's ingenuity and skill. This relationship likely functioned as a quid pro quo: Pacioli served as Leonardo's personal mathematics tutor, while Leonardo

[1] Tennenbaum, 'Luca Pacioli's 1498 On the Divine Proportion', p. 2. Located about 60 miles east of Florence, in the 15th century San Sepolcro was under the control of the Florentine Republic.
[2] Nicholl, 'Leonardo: Flights of the Mind', p. 307-309; Glori, The Cartouche of the Double Portrait of Luca Pacioli and Pupil, p. 26;

acted as Pacioli's personal illustrator and maestro. Pacioli's admiration for Leonardo's artistic and intellectual achievements is further evidenced by his mention of Leonardo's 'Paragone', a discourse comparing the merits of painting and sculpture. This relationship between the two Renaissance figures highlights a fruitful exchange of knowledge and skills, blending art, mathematics, and intellectual inquiry.

In the initial chapter of 'De Divina Proportione', Luca Pacioli, after briefly acknowledging all the distinguished attendees of a scientific disputation held at Ludovico Sforza's ducal palace, devotes almost an entire page to introducing Leonardo da Vinci, who was among the guests. Pacioli describes Leonardo as a Florentine compatriot, renowned among a group of discerning architects, engineers, and inventive creators.

Pacioli extols Leonardo, stating,

> "Among the company of most discerning architects, engineers and assiduous inventors of new things, Leonardo da Vinci, our Florentine compatriot, who, with every cast, sculpture and painting, confirms his surname.[3] As with the admirable and stupendous equestrian statue, -- dedicated to the sacred memory of your glorious father --, whose height from the top of the head to the level ground is 12 braccia i.e., 36 times as much as the line AB given here in the margin, and in all its enormous mass amounts to about 200,000 *pounds*, where each *pound* is made up of 12 common ounces which has nothing to envy from the work of Phidias and Praxiteles in Monte Cavallo; and of like excellence, the beautiful painting of the ardent desire of our Salvation, painted by his hand in the worthy and sacred refectory for receiving both bodily and spiritual nourishment, in the sacred temple of the Graces[4] -- to whom today, Apelle, Miro, Policretus, and others, it is clear, must give the palm.[5] And not satiated with all these, he expects to bring his invaluable work on locomotion, on percussion, weights, and all

[3] The name Vinci is from word *vincere*, which means to win, conquer or excel.
[4] This is the first reference to Leonardo's Last Supper painted in Santa Maria delle Grazie which Pacioli calls 'Temple of Graces'.
[5] The expression 'giving the palm' is a metaphor for victory. It has a double meaning, the first being from ancient Rome when victors in a contest were awarded a palm leaf by a magistrate or judge, the second, from the decision handed down by a judge in a dispute, when he shows his upward facing palm to the victor. We see this most often depicted in Last Judgement scenes where Christ displays his right hand, upward facing open palm to the 'saved', but hides his lefthand downward facing palm to the damed. Leonardo depicted this image of Christ in his Last Supper, albeit reversed. More on tis in my forthcoming study.

kinds of forces, that is, accidental weights, to a worthy conclusion - -- having already with all diligence finished his valuable book on painting and human motion."[6]

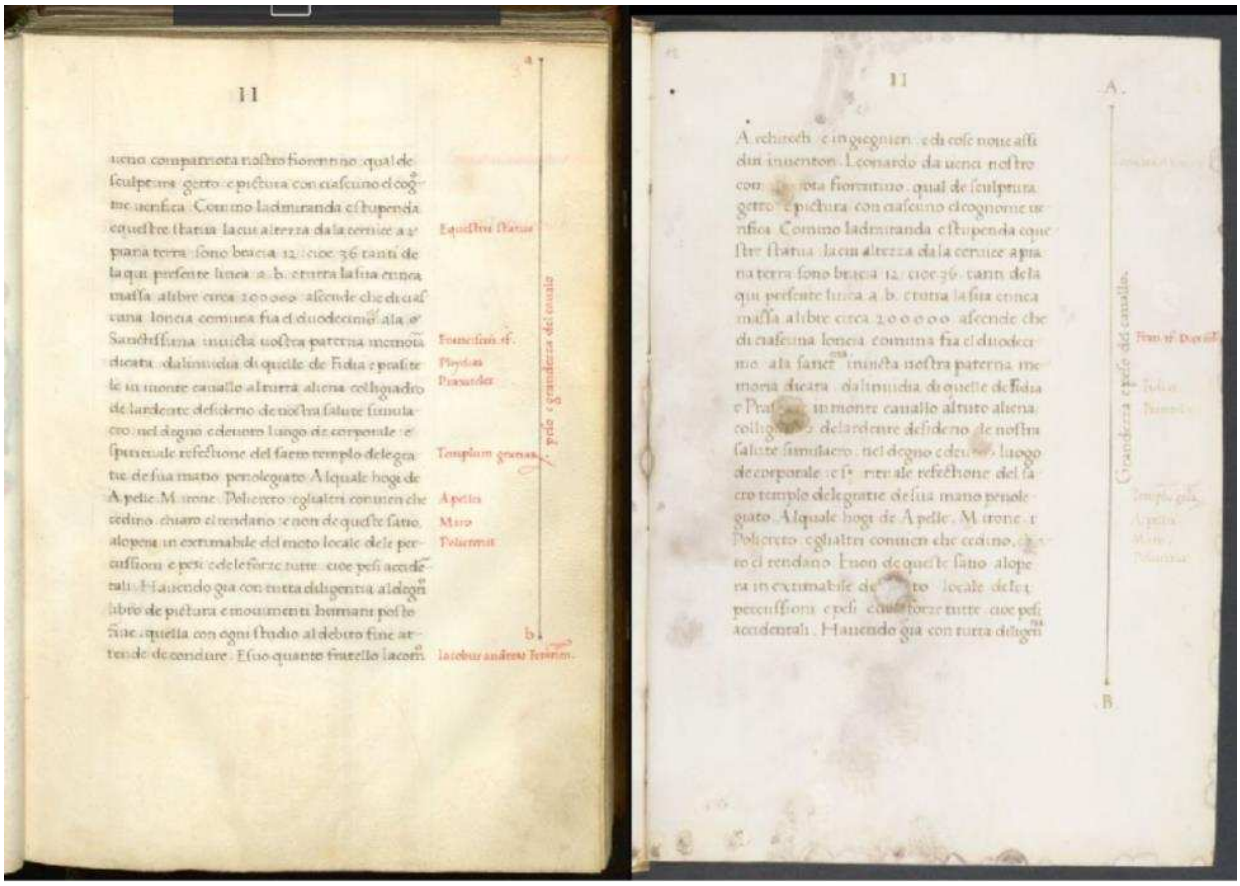
Pacioli's introduction of Leonardo, while acknowledging him as part of a group of architects, engineers, and inventors, singles him out as the epitome of Florentine sculptors and painters. This is particularly significant praise, considering Florence's global reputation as a hub of architectural and artistic innovation and skill. Pacioli then provides two examples of Leonardo's exemplary work: the Sforza monument, symbolizing Sforza power, and the Last Supper, reflecting Sforza spirituality. Both works are intimately connected to Ludovico Sforza's heart and legacy, showcasing Leonardo's profound impact in the realms of artistic and spiritual expression.

Pacioli's line AB and Leonardo's braccio

My perspective on the proportional canon of the Last Supper underwent a significant shift after delving into Pacioli's initial remarks about Leonardo's expertise and patronage. The revelation was so clear and immediate that I was puzzled why it had remained overlooked for such an extended period. To understand this better, let's revisit what Pacioli specifically noted about the Sforza horse:

"The height of the horse, from the top of its head to the level ground, measures 12 braccia, which is to say, 36 times the length of the line AB that is provided here in the margin."

[6] Leonardo was at work on his 'Treatise on Painting' which was never completed.



Da Divina Proportione
(1498, Milan)

Da Divina Proportione
(1498, Geneva)

Figure 1

In the margin of the folio that references Leonardo, the Sforza horse, and the Last Supper, there lies the first illustration in the entire book: a line labeled AB (Figure 1). This line can be deduced to be one-third of a braccio.[7] This calculation stems from the fact that the Sforza horse, as designed by Leonardo, stands 12 braccia high, and Pacioli mentions that line AB is 1/36th of the horse's height. Therefore, if the horse is 12 braccia high, then line AB, being 1/36th of that, must logically be one-third of a braccio, since 36 divided by 12 equals 3. Consequently, according to Pacioli, the braccio used by Leonardo for the Sforza horse (and, as I will demonstrate, for the

[7] The "braccio," often used in historical Italian measurements, was a unit of length. Its name literally means "arm" in Italian, reflecting its origin as a measurement based on the human body, similar to the "foot" or "yard" in English measurements. The length of a braccio varied from city to city in Italy, as there was no standard measurement system across the country during the Renaissance and earlier periods.

Last Supper) is precisely three times the length of the line AB, as hand-drawn in the folio's margin.

This led me to wonder, "Has anyone ever precisely measured line AB in the folio, and if not, how can I access the original manuscript to measure it myself?" This is where eCodices, the Virtual Manuscript Library of Switzerland, comes into play. They provide a high-resolution, fully digitized copy of the original manuscript housed in Geneva, complete with detailed measurements of the manuscript's page layout and the faintly inked margins. [8]

While this isn't a direct on-site measurement of line AB, it allowed me to compare the visibly longer, unknown length of line AB to the visibly shorter, known length of the inked margins on the same page, establishing their relative proportions (line AB : margin). Remarkably, if we assign the margin a length of 1.00, the measured length of line AB is exactly 1.1245. Therefore, line AB measures 195.666 millimeters, given that the length of the margin is reported by the Library of Geneva to be 174 millimeters.[9] By referencing these hand-drawn inked margins, I determined that the digitally measured length of line AB in Pacioli's original manuscript is 195.666 mm. Since Pacioli states that line AB is one-third of Leonardo's braccio, the full braccio length is thus three times line AB, equating to 587 mm.[10] This calculation means Leonardo's horse sculpture would stand at an impressive height of 12 x 587 mm, or 7.0438 meters (approximately 23.109 feet), making it a truly massive work of art.

To confirm this discovery, my immediate focus shifted to locating another known manuscript of 'De Divina Proportione', produced concurrently for Galeazzo Sanseverino. Sanseverino was not only the Duke's second cousin and son-in-law but also the Captain General of his military forces.[11] This particular manuscript, currently housed in Milan, is believed to have been crafted by Giovanni Battista Lorenzi, the same scribe responsible for the Geneva version. Intriguingly, the manuscript presented to Sanseverino is of higher quality than the one in Geneva given to

[8] Genève, Bibliothèque de Genève, Ms l.e. 210 indicates "Page layout: Barely visible ink ruler, justification: 174 x 104 mm, long lines, 25 lines per page (f. 3r - 11r , table of contents) and 23 lines per page" found at https://www.e-codices.unifr.ch/en/description/bge/le0210/Jeger

[9] 174 mm x 1.1245 = 195.666 mm.

[10] Throughout Italy a braccio was anywhere between 550 to 650 millimeters or about 22 to 26 inches. However, this measurement could differ significantly depending on the region or period.

[11] Galeazzo was the son of Roberto Sanseverino, who was the son of Elisa Sforza, who was the sister of Francesco Sforza, Ludovico Sforza's father.

Ludovico Sforza, leading some to speculate that Leonardo himself may have been more directly involved in the creation of the polyhedrons in this copy.[12]

After extensive research, I managed to locate a very high-resolution digital version of the Milan manuscript. By comparing and verifying the identical size of the barely visible margins with those in the Geneva copy, I was able to confirm that line AB in the Milan manuscript is indeed exactly the same length as in the Geneva version, measuring precisely 587 mm. This confirmation not only reinforced the initial revelation but also highlighted the meticulous consistency in the production of these manuscripts, further underscoring the interconnectedness of Leonardo's artistic endeavors with Pacioli's mathematical works.

The braccio at Santa Maria delle Grazie and San Lorenzo

The question arises: Did Leonardo da Vinci apply the 587mm braccio measurement in other works? A logical place to start investigating is another significant project he was involved in at the same time, namely 'The Last Supper', painted on the north wall of the refectory at Santa Maria delle Grazie. Due to the imperfections of human construction, the impacts of war, etc… this wall and its adjacent walls are not 'perfectly' squared, resulting in a slight variation in the painting's width, ranging from 8792 to 8808 millimeters.[13] This measurement aligns with the pre-war survey conducted in 1937 by Agnoldomenico Pica and is further corroborated by both restoration efforts and recent high-resolution digital imaging.[14]

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[12] See Marani, 'Leonardo's Cartonetti for Luca Pacioli's Platonic Bodies'. Although both copies were transcribed by the same amanuensis Giovanni Battista Lorenzi , it is possible that the manuscript presented to the duke (now found in Geneva) was thought by Pacioli and Leonardo to be a mediocre copy of the original (now lost), therefore they were both more 'hands-on' involved in the next production for Sanseverino, with some proposing that Leonardo, protecting his reputation, painted the illustrations in the copy for Sanseverino himself (instead of leaving it to Lorenzi or one of his assistants). Indeed, the illustrations in the Ambrosiana copy are notably superior to the copy in Geneva, and even the formatting of marginal notes on each page of the Ambrosiana display Lorenzi's improvement over the marginal notes in Geneva.

[13] Simply age in general will cause a building to 'lean'. See Landrus, M. (2019). Rational Estimates for Irrational Problems: Proportional Geometry in the Work of Leonardo da Vinci. In: Cocchiarella, L. (eds) ICGG 2018 - Proceedings of the 18th International Conference on Geometry and Graphics. ICGG 2018. Advances in Intelligent Systems and Computing, vol 809. Springer, Cham. https://doi.org/10.1007/978-3-319-95588-9_3

[14] Barcilon and Marani, Leonardo: L'ultima cena, p. 166; https://www.haltadefinizione.com/. Depending on the elevation of the measurement, a slight variance of +/- 8 mm in width can be measured using the Haltdefizione application. This is likely because of the ravages of time, weathering and/or noticeable trauma to the corner which joins the west and north walls. For instance, the width measured from top of the painting measures 8808 mm, the bottom 8795 mm and the middle 8792 mm.

It is understood that Leonardo initially painted the lunettes and the accompanying architrave, likely progressing downwards to complete the rest of the painting.[15] For my analysis, I chose to use the width of 8805 mm, which corresponds to the measured width of the outside-bottom of the architrave, a section Leonardo likely painted before the interior depiction of Christ's 'upper room'.

Intriguingly, when comparing the 587 mm braccio from 'De Divina Proportione' to the width of 'The Last Supper', there is an exact division into 15 braccia, for $\frac{8805\ mm}{587\ mm} = 15.000$. This precision leads me to hypothesize that the width of the refectory's north wall was either originally designed to be, or very close to, 8805 mm, or that Leonardo perceived it as such. A measure of 8805 mm has an exceedingly miniscule discrepancy from Pica's report of 8800 mm, for $\frac{8805-8800}{8800} = 0.056\%$, so it is probable that Pica rounded down from 8805 to an even 8800.[16]

To further substantiate that the measurement of Pacioli's line AB and the dimensions of the north refectory wall are not merely coincidental, it's insightful to examine the measurements in other parts of the Santa Maria delle Grazie complex, specifically the church's tribune/dome and the adjacent 'Cloister of the Dead', which shares its western wall with the refectory. It's important to note that both the refectory and the Cloister, along with other structures like the kitchen, dormitory, chapter room, and later, the library, were part of the original construction

[15] See Barcilon and Marani; Leonardo Last Supper p. 407-9

[16] A. D. Pica, I Monumenti italiani, rilievi raccolti a cura della Reale Accademia d'Italia, X, Rome I939; When observing ground plan measurements for spaces larger than a few meters, Pica preferred to round up or down (in centimeters) to the nearest 20th of a meter, for his measures always ends in 0 or 5. In his twenty-three measurements of the 'Planimetria Generale Allo Stato Attuale' and 'Particolare Della Planimetria', twenty of his measurements end in 0, whilst the remaining three end in 5. For example, if Pica's actual measure of the refectory was 8805 millimeters, then Pica would convert this to 8.805 meters and then round down to 8.80 meters and not up to 8.81 meters, because 8.81 meters is not a 20th of a meter (it is 81/100th of a meter). 8.80 and 8.85 represent 16/20th and 17/20th of a meter respectively, and since 8.805 is much closer to 8.80 than 8.85, Pica went with 8.80. It is likely that he did the same with the measure of the interior square of the Cloister of the Dead and the dome of the Tribune. If the actual measure of the interior square of the Cloister is 27208 millimeters (as I suppose, for 27208 / 8805 = 3.0901), then Pica would convert this to 27.208 meters and then minutely round down to 27.20 meters (4/20th of a meter) and not up to 27.25 meters (5/20th of a meter), for 27.20 is closer to 27.208. The same goes for the Tribune, for if the width of the Tribune measures 17610 millimeters (as I suppose, for 587 x 30 = 17610), then Pica would convert this to 17.610 meters and then minutely round down 1 centimeter to 17.60 meters (12/20th of a meter) and not up 4 centimeters to 17.65 meters (13/20th of a meter). However, if the width of the Tribune was 17630 millimeters, then Pica would convert it to 17.630 meters and then obviously round up 2 centimeters to 17.65 meters (being 13/20th of a meter) and not down 3 centimeters to 17.60 meters.

initiated by Solari in the 1460s. Meanwhile, the church's tribune was a part of an expansion project inspired by Brunelleschi, started in the early 1490s.[17]

A closer look at Agnoldomenico Pica's survey of the Cloister reveals that the interior square of the cloister measures 27200 mm in width (Figure 2).[18] Although Pica did not measure the exterior square of the cloister, which includes its four-sided porch, his precise drawings indicate that its width is exactly four times that of the refectory, which -- when using the 587 braccio -- would make it precisely 35220 mm (60 braccio), for $\frac{35220\ mm}{8805\ mm} = 4.000$.[19]

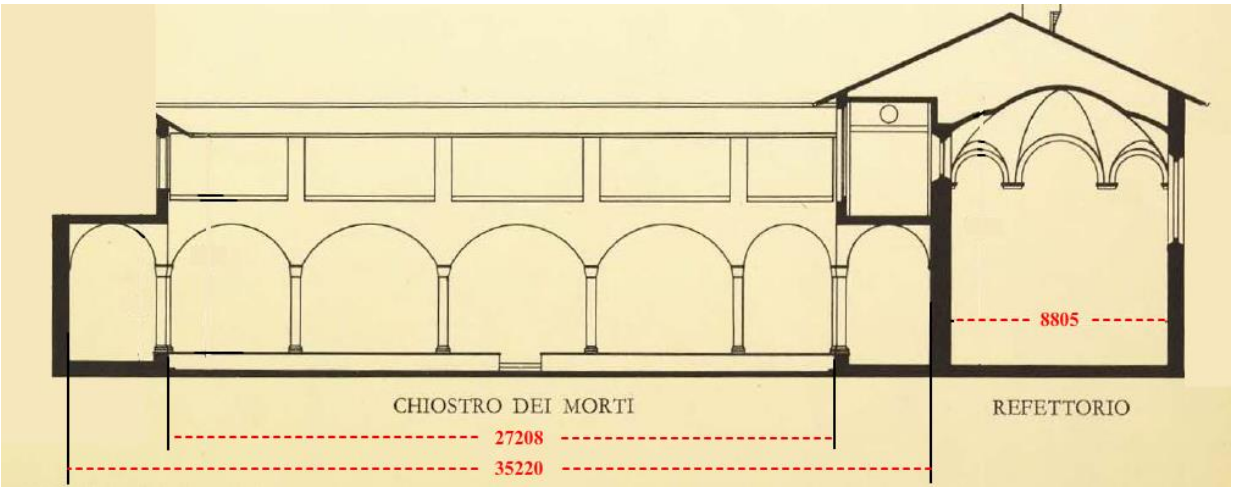


Figure 2

It's also interesting that Pica reported the width of the church's interior dome and tribune – which was constructed more than 20 years after the refectory and cloister -- to be 17600 mm, or precisely twice the width of the refectory for, $\frac{17600\ mm}{8800\ mm} = 2.000$ (Figure 3). My supposition is that since the refectory is actually 8805mm wide, the actual width of the dome (being twice the width of the refectory and half the width of cloister's exterior square) is 17610 mm and that again Pica simply rounded down to nearest 20th, i.e. 17.60. What this means is that the radius of 'Bramante's' dome is the same as the width of the refectory while it's diameter is half the width

[17] Marani, 'il Cenacolo, Guide to the Refectory and Church of Santa Maria delle Grazie', p. 49-75; Aiello and Bolognesi, 'Reliving History: The Digital Reconstruction of the Convent of Santa Maria Delle Grazie in Milan;
[18] See note 15.
[19] For the purpose of this study, I am supposing that the actual width of the cloister's exterior square is 35220 mm, for 8805 x 4 = 35220, and because I extrapolated this measurement from Pica's 1937 scale and not his on-site measurement, I am only considering the result as 'highly probable' and not certain.

of the cloister's exterior square, so in this way its proportional measure serves as a type of 2:1 geometrical mediator between the wider Cloister of the Dead and the narrower Refectory.[20]

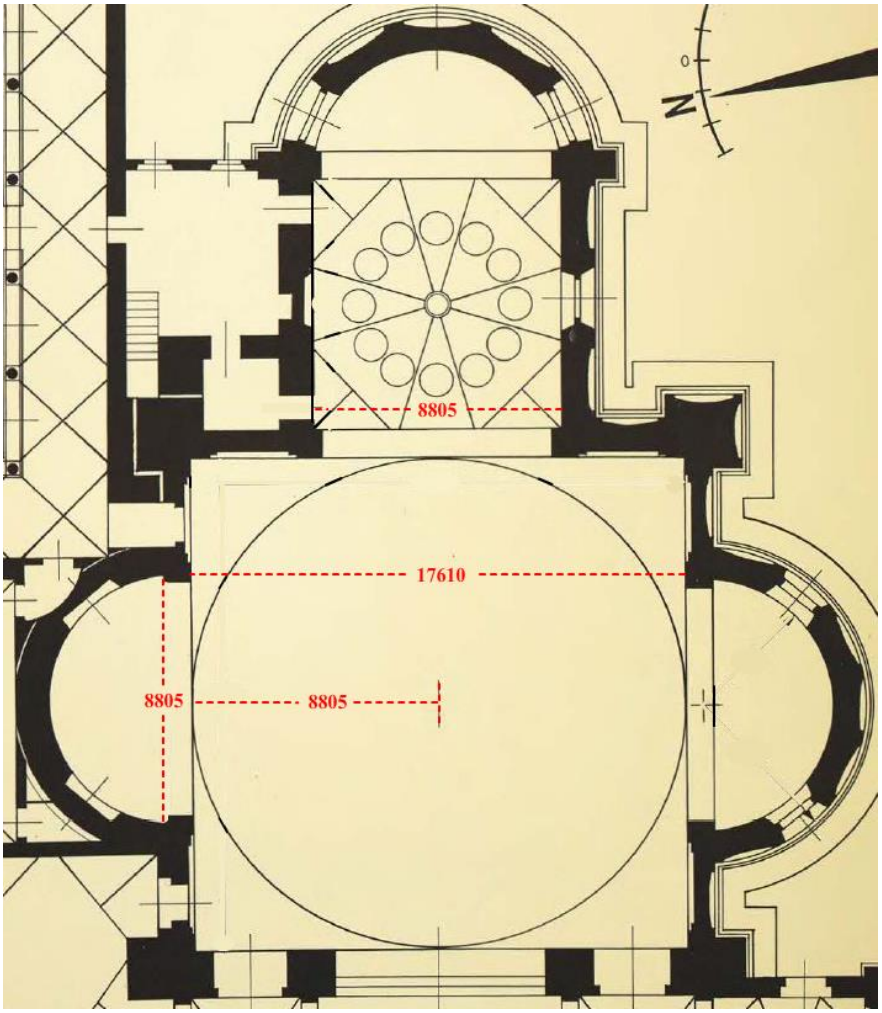


Figure 3

Equally remarkable is the discovery that the distance from the center of the refectory's north wall (the vanishing point in Leonardo's 'The Last Supper') to the portal of the church's nave is precisely twice the length of the Cloister's interior square. Furthermore, this distance is exactly equal to the distance from the church's portal to the High Altar, situated directly beneath the dome's apex, creating a perfect 45-45-90 right triangle (Figure 4). The architectural and theological significance of this symmetry is profound. It echoes Brunelleschi's groundbreaking

[20] The ratio of the Cloister to the Dome is 2:1 and the ratio of the Dome to the refectory is 2:1, making the ratio of the Cloister's width to the refectory's width 4:1.

concept of the ‘eye of perspective’. The equal distance from the church’s portal to both Leonardo’s strategically elevated Eucharistic vanishing point in the refectory and the high altar under the dome's eye serves as a symbolic reflection.[21] It mirrors the Eucharist being celebrated on the elevated high altar, reinforcing the spiritual connection and architectural harmony within the sacred space.

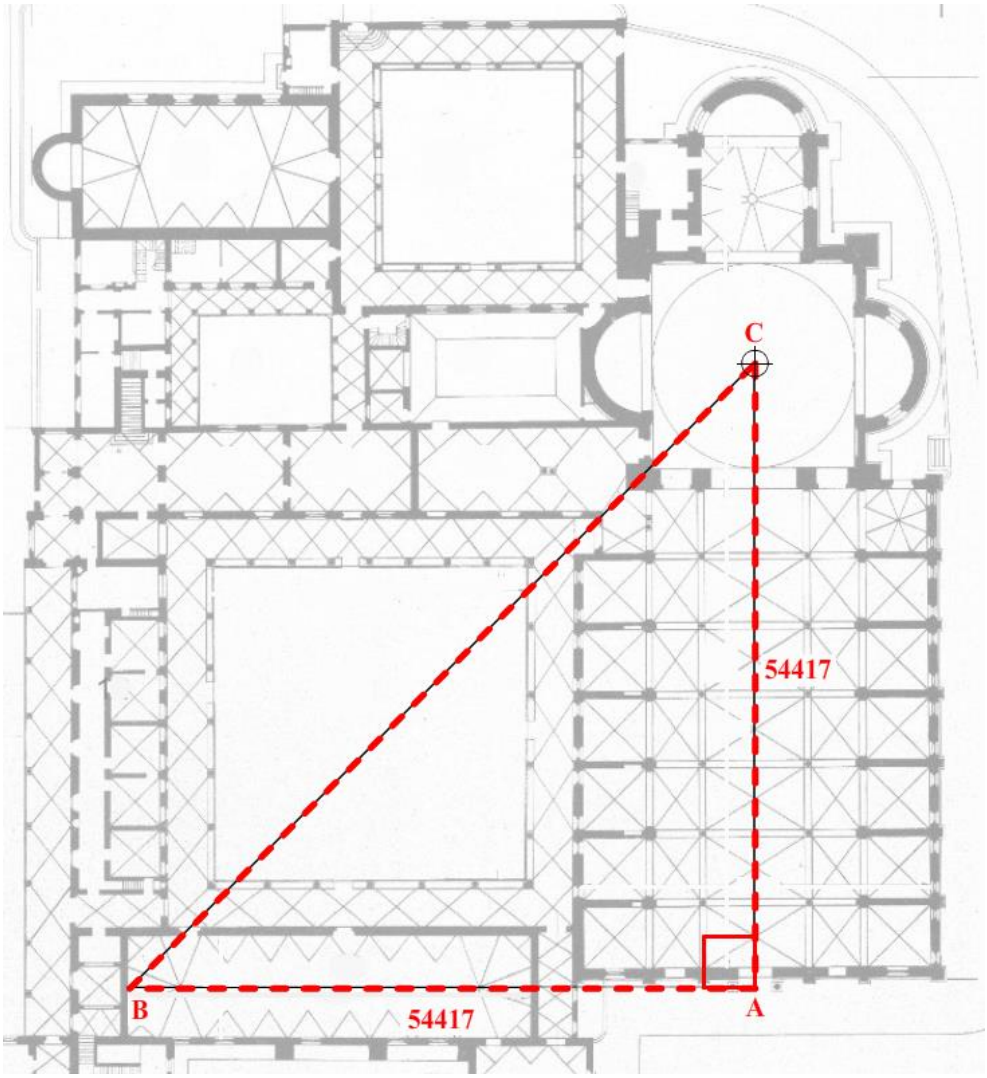


Figure 4

[21] In my next paper I’ll demonstrate the how the vanishing point is thought to represent both the painter and the painting’s ‘eye’.

Hence, it becomes evident that the wall on which Leonardo da Vinci painted 'The Last Supper' is in precise proportional relation not only to the exterior square of the Cloister of the Dead but also to the critically significant dome/tribune — the most sacred part of the convent where the Eucharist is celebrated.[22] This alignment provides compelling evidence that the architect responsible for the tribune was theologically and architecturally intentional in integrating his design with the convent's preexisting structures, where the monks, emulating Christ, partook in supper and awaited resurrection.[23]

But what of the interior square of the cloister? Was it of significance to the designers or to Leonardo himself? Intriguingly, and a point we will explore in more detail later, is the fact that the width of the cloister's interior square relative to the width of the refectory's north wall mirrors the ratio of the refectory's north wall to the rear wall of the painted 'upper room' that Leonardo depicted in perspective, for $\frac{27200\ mm}{8805\ mm} \approx 3.090 \approx \frac{8805\ mm}{2849.35\ mm}$ which those of us familiar with the Golden Ratio will instantly notice is half of 6.180.

Therefore, comparing the width of the Cloister to the width of the refectory we observe a nearly exact 4:1 ratio, i.e., $\frac{35200\ mm}{8805\ mm} = 3.997$, a percentage error of only 0.057%. This means that if the refectory is 15 braccia wide, then the Cloister spans 60 braccia, suggesting that Guiniforte Solari constructed both the refectory and the Cloister concurrently, adhering to the 587 mm braccio.[24] This leads to a fascinating question: How did Solari come to adopt the 587 mm braccio in his construction?

Milan, Florence and San Lorenzo

The architectural and historical landscape of Renaissance Italy, particularly in the relationship between the Basilica di San Lorenzo in Florence and Santa Maria delle Grazie (SMG) in Milan, is a

[22] Amazingly, featuring the number 5, the perfectly square cloister has 25 porticoes, which is 5^2. What has perplexed scholars is why the porticoes on North, West, South and East are segmented in 6 + 6 + 8 + 5 = 25 respectively. I only note here that 5 is the most significant rational number for Pacioli's arithmetic concerning the Divine Proportion, and the ratio of 8 : 5 is not only part of the Fibonacci sequence, but its sum (13) is a number prominently featured in sermons about the Last Supper with 13 participants and Pacioli's 13 divine effects of the Divine Proportion. In this way both the number 12 (6 + 6) and 13 (8 + 5) are featured in the cloister.

[23] Steinberg, Leonardo's Incessant Last Supper, p. 145

[24] It is mostly undisputed that Solari was the main architect behind the design and construction of Santa Maria delle Grazie during the 1460s and 70s.

testament to the era's intricate blend of religious tradition, political power, and artistic innovation. This relationship, deeply influenced by the Medici family's patronage and the Florentine architectural advancements, is embedded in the cultural and religious identity of these cities, reflecting a complex interplay of influences.

San Lorenzo, renowned as Florence's oldest documented church and its inaugural cathedral, occupies a foundational position in the city's religious heritage.

San Lorenzo, consecrated and dedicated to Saint Lawrence in 393 AD by Saint Ambrose, the esteemed Archbishop of Milan and a prominent Church Father, played a pivotal role in establishing a lasting ecclesiastical connection between Florence and Milan. Initially known as 'ecclesia Ambrosiana', this dedication by Saint Ambrose created a profound bond between the two cities.[25] The consecration of San Lorenzo by Saint Ambrose, further marked by a series of reported miracles attributed to him, went beyond forging a vital religious link. It endowed San Lorenzo with a substantial spiritual legacy, cementing its status as a foundational part of Florence's religious community.

The Feast of San Lorenzo, celebrated on August 10th, continues to be a major observance in Florence and stands as a tribute to this legacy. Characterized by a blend of religious ceremonies and cultural celebrations, the feast day encapsulates the enduring Christian traditions of Florence and its unwavering reverence for the cathedral and its patron Saint Lawrence. This annual celebration not only honors the city's rich spiritual heritage but also acts as a powerful reminder of the historical and spiritual connection between Florence and Milan.

In the early 15th century, San Lorenzo became closely linked to the Medici family, particularly through Giovanni di Bicci de' Medici and his son, Cosimo. Chosen as their family church only yards away from their new residence on Via Larga, San Lorenzo was a symbol of the Medici's growing influence in Florence. The Medici's involvement with San Lorenzo coincided with a significant event: the commencement of the church's reconstruction, which began on the Feast of San Lorenzo in 1421. This project, heavily funded by Giovanni di Bicci de' Medici, was more

[25] Tacconi, M. (2005). Cathedral and Civic Ritual in Late Medieval and Renaissance Florence: The Service Books of Santa Maria Del Fiore. United Kingdom: Cambridge University Press, p. 60-4;
Morillo, S., Morillo, S. R., North, W., Taylor, A., Jones, A. T., Urbanski, C. (2008). The Haskins Society Journal 19: 2007. Studies in Medieval History, p. 66-8

than just a renovation; it was a statement of the family's burgeoning power and their dedication to art and architecture.[26]

Entrusted to Filippo Brunelleschi, a master architect of the time, the reconstruction of San Lorenzo was a pivotal moment in both the church's and Florence's history. Brunelleschi's work on the Medicean sacristy/mausoleum, known as the old sacristy, was groundbreaking. His innovative designs and techniques not only transformed San Lorenzo but also heralded a new era in architectural history. Brunelleschi's innovations at San Lorenzo profoundly influenced the architectural developments in Sforza's Milan.[27] This impact was notably evident in the construction of the tribune at Santa Maria delle Grazie (SMG) and the Portinari Chapel at Sant'Eustorgio, where Brunelleschi's pioneering techniques at San Lorenzo were echoed and adapted in Dominican structures.[28]

Medici and Sforza settle a debt

In the broader context of the Florence-Milan alliance, the close relationship, and roles of Pigello Portinari and Gaspare Vimercati were instrumental. After the Battle of Anghiari, Portinari (raised by Cosimo de' Medici), and Vimercati (a close confidant of Francesco Sforza) played a crucial role in reinforcing the alliance between these two powerful cities. Portinari served as the Medici's representative in Milan and was the executor of Vimercati's will. This alliance, underpinned by financial interdependencies, saw the Medici family providing continuous and exorbitant financial support to the Sforza, ensuring military protection for Florence in return.[29]

Gaspare Vimercati, a distinguished count, Milanese military leader and trusted confidant of the Sforza family, played a pivotal role in the establishment of Santa Maria delle Grazie (SMG). In 1462, Vimercati embarked on a crucial mission to Florence with the aim of settling the Sforza's

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[26] Saalman, H. (2010). Filippo Brunelleschi: The Buildings. (n.p.): Pennsylvania State University Press, p. 108-13

[27] WATERHOUSE, E. K. (1973). THE RENAISSANCE IN MILAN BEFORE BRAMANTE AND LEONARDO. *Journal of the Royal Society of Arts*, *121*(5202), 381–391. http://www.jstor.org/stable/41371078

[28] Marani, 'il Cenacolo, a Guide to the Refectory and Church of Santa Maria delle Grazie', p. 59-60, 78-9; Covini, 'Il Fondatore delle Grazie Gaspare Vimercati, Gli Sforza e Gli Altri Benefattori', in Memoriedomenicane; il convent di santa maria delle grazie a Milano, p. 63-67; Steinberg, Leonardo's Incessant Last Supper, p. 145-47. Pedretti remarks that that Santa Maria delle Grazie was for the Sforza what San Lorenzo was for the Medici, a family mausoleum. That the Sforza duke sought to emulate the Medici of Florence is well known, and surely the duke would have known that the tomb of Cosimo Medici in San Lorenzo, completed by 1467, lies at the foot of the high altar, directly under the dome.

[29] De Roover, R. (1999). The Rise and Decline of the Medici Bank, 1397-1494. United States: Beard Books, p. 261-4;

massive outstanding debts to the Medici. This mission was more than financial negotiation; it represented a significant evolution in the ties between Milan and Florence. Successfully resolving the debt issue and bringing the flow of Medici money back into Milan, Vimercati acted under the directive of Cosimo de' Medici to establish SMG as an Observant Dominican convent in Milan.[30] This move was a direct reflection of Cosimo's deep-seated religiopolitical interest, particularly his desire to replicate in Milan the presence of an Observant Dominican institution akin to his beloved San Marco in Florence.

The foundation of Santa Maria delle Grazie (SMG) in Milan, laid in 1464, represents a significant chapter in the architectural narrative of the Renaissance, closely following the initiation of the Brunelleschi-inspired Portinari Chapel in Milan's first Dominican convent, Sant'Eustorgio.[31] Guiniforte Solari, the architect behind SMG, is suggested as the builder of Portinari's chapel, establishing a tangible architectural connection to the work of Filippo Brunelleschi at San Lorenzo.[32] This link is not merely theoretical but is manifest in the striking architectural parallels between the two structures.

[30] Covini, 'Il Fondatore delle Grazie Gaspare Vimercati, Gli Sforza e Gli Altri Benefattori', in Memoriedomenicane; il convent di santa maria delle grazie a Milano, p. 63-67; The alliance between Florence and Milan after the Battle of Anghiari was strong. Through continual loans, the new duke Francesco Sforza enjoyed the influx of Medici money into Milan and Cosimo enjoyed the protection that the Sforza army offered Florence. As such Portinari not only became a permanent resident in Milan but had very close diplomatic and financial ties with both the Duke and the founder of Santa Maria delle Grazie, Gaspare Vimercati, a powerful Milanese military leader and close confident to the Sforza.[30] By 1462, the Sforza had various unpaid debts to the Medici totaling a massive 50,000 florins, putting not only all loans from Cosimo to the Duke on hold, but also their strong alliance on volatile ground. With his celebrated arrival, Vimercati took up residence in the Medici palace as a personal guest of Cosimo. His mission must have been successful, for in the following months the remittances of money from Florence to Milan resumed and by October 1463 another large loan of 40,000 ducats arrived in Milan, 13,000 of which went directly to Vimercati 'to collect other pledges for those who are lost'. This correlates well with Gattico's later account that Cosimo also persuaded Vimercati to build an Observant Dominican convent -- like 'his' personal convent of San Marco -- in Milan, and so Covini sees in this 'pledge of money' by Cosimo -- and his close ties to Portinari -- the foundation of the Dominican convent Santa Maria delle Grazie (SMG) by Vimercati.

[31] Marani, 'il Cenacolo, a Guide to the Refectory and Church of Santa Maria delle Grazie', p. 49-53. The trend with most scholars today is that Solari (not Filarete) designed the Portinari chapel thus introducing, by way of Pigello Portinari, the Brunelleschian form of architecture to Milan. Like the Pazzi Chapel in Florence, the Portinari Chapel in Milan was designed to emulate the Old Sacristy in San Lorenzo.

[32] Ackerman, J. S. (1994). Distance Points: Essays in Theory and Renaissance Art and Architecture, p.60-1.

Santa Maria dell Grazie, a measured mirror of San Lorenzo

San Lorenzo and Santa Maria delle Grazie (SMG) exhibit a remarkable synchronization in their dimensions and orientations, meticulously adhering to their respective liturgical axes. The alignment of each structure's axis demonstrates an impressive precision, deviating by a mere 0.48° from one another.[33] This level of accuracy in their alignment suggests intentional design rather than random coincidence.

The chosen orientation for SMG is particularly striking. When presented with the option to align SMG to an azimuth of exactly 90°, in line with True East-West as commonly seen in many basilicas of that era, architect Guiniforte Solari made a notable decision. He opted for an orientation of 104.92°, carefully mirroring the orientation of San Lorenzo, which is set at 104.44° (Figure 5). The reason behind San Lorenzo's specific liturgical axis orientation is clear: it provides a perfect view of the sunset through the church's rear window on August 10th, the Feast of San Lorenzo.[34] This deliberate alignment of SMG, echoing the orientation of San Lorenzo, signifies Solari's purposeful architectural dialogue with, and homage to, the esteemed Brunelleschi, thus reflecting a deep consideration of San Lorenzo's significance.

The precision in the dimensions and length of the braccio shared between these two structures further underscores San Lorenzo's influence upon Solari.[35] The interior length of San Lorenzo's east-west axis, at 78.80 meters, is exactly replicated in the exterior length of SMG's east-west

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[33] Observations and computations were primarily made using Google Earth Pro and Starry Night Pro. On the accuracy of using GoogleEarth Pro (GEP) heading information for archaeoastronomy see William F. Romain 'Notes on the Accuracy of Google Earth Pro Heading Information (2022)'. The heading of San Lorenzo is 284.4 degrees, and the heading of SMG is 284.9 degrees, making the difference between the two a virtually indistinguishable 0.50 degrees or 0.175%. In fact, if we factor in the fact that SMG in Milan is located at 45.465 latitude and San Lorenzo in Florence at 43.774 latitude, then the minuscule difference makes sense if we consider that the builder of SMG was aware of the sun's 3.75 degrees altitude at San Lorenzo on August 10th, 1463.

[34] On August 10th, 1463, at an azimuth of 284.4° (which 180° 'opposite' of 104.4°) the setting sun can be viewed at an altitude of 3.75°. The altitude of 3.75 degrees is significant, for it is at this very low altitude that the setting sun at both San Lorenzo in Florence and SMG in Milan hangs just above the horizon. In the 15th century on August 10th during evening mass, the low sun would shine directly through the portal of their 'east-west' axis and illuminate the nave. At SMG it is likely that the low sun would expand across the nave and briefly terminate at the altar, thereby illuminating it. At San Lorenzo, since the portal and altar are rotated 180 degrees from SMG, the same phenomena would be observed, however the sunlight would not come from the window above the main entranceway in the east, but from the opposite rear window behind the main altar in the west. The effect at San Lorenzo would perhaps be more intense than SMG if the window contained a stained-glass image of Saint Lawrence or a crucifix was positioned directly in front of it.

[35] "San Lorenzo (church). Plan.; Filippo Brunelleschi; 1419". https://quod.lib.umich.edu/u/ummu2ic/x-ls017348/ls017348. University of Michigan Library Digital Collections.

axis (Figure 6). Similarly, the interior measure of San Lorenzo's north-south axis, at 31.80 meters, matches precisely with the exterior measure of SMG's north-south axis. This perfect congruence in both length and width indicates that SMG was meticulously designed to fit within the interior dimensions of San Lorenzo.

Furthermore, the fact that the axes of both structures are directional mirrors of each other introduces an additional element of intentional design. This meticulous azimuth alignment, coupled with the corresponding measurements, strongly indicates that Santa Maria delle Grazie (SMG) was architecturally conceptualized as an echo of San Lorenzo. A hair smaller in scale, SMG not only replicates San Lorenzo's structural proportions but also 'reflects' its directional orientation, serving as a homage to the ancient Florentine basilica. This deliberate mirroring goes beyond mere structural mimicry, embodying a deeper reverence and connection to San Lorenzo's spiritual legacy and Medici patronage.

Considering the striking architectural similarities between Santa Maria delle Grazie (SMG), serving as the mausoleum for the Sforza family, and San Lorenzo, the final resting place of the Medici, it is a reasonable hypothesis that Guiniforte Solari, the architect responsible for SMG, drew inspiration from the Medici's San Lorenzo, particularly in terms of measurement systems. This hypothesis is particularly focused on Solari's potential use of the 587 braccio measurement, a unit evident in the interior design of San Lorenzo.

In an insightful study by Trachtenberg, an on-site survey of San Lorenzo conducted by Herzner revealed key measurements that align with this hypothesis.[36] The survey found that the interaxial distance between columns in the nave of San Lorenzo was set at 12.18 by 6.58 meters. In opposition to Burns, Trachtenberg hypothesized that a braccio of 586 mm was in use by Brunelleschi at San Lorenzo giving a measure of 20.75 by 11.228 braccio,[37] but his computation

[36] Trachtenberg, Building and Writing S. Lorenzo in Florence: Architect, Biographer, Patron, and Prior, p. 150-151, 170-171, citing ("Zur Baugeschichte," 105)

[37] Unlike today there was no universal standard of measure in cities, much less regions. Therefore, it is quite common to find small variances between architectural plans across short time periods. It is well known that in 15th century Florence the braccio varied between 583 and 588 mm, which makes it notoriously difficult to do a precision comparison between structures. Measurements for this study were done by taking on-screen measurements using a digital caliper from online publications of site surveys which provided a scale of measurement. See, "San Lorenzo (church). Plan.; Filippo Brunelleschi; 1419". https://quod.lib.umich.edu/u/ummu2ic/x-ls017348/ls017348. University of Michigan Library Digital Collections; Burns, H. (1979). San Lorenzo in Florence before the Building of the New Sacristy: An Early Plan. *Mitteilungen Des Kunsthistorischen Institutes in Florenz*, *23*(1/2), 145–154; Cohen, M. A.

is slightly inaccurate, for a measure of 12.18 meters divided by a braccio of 586 millimeters would result in a long interaxial measure of 20.78 braccio and not 20.75 braccio ($\frac{12.18\ m}{586\ mm} = 20.78$). However, if we theorize that Brunelleschi used a 587 mm braccio (which is equivalent to 20 soldi) and not a 586 mm braccio, then not only does the long interaxial distance calculate to exactly 20.75 braccio, for $\frac{12.18\ m}{587\ mm} = 20\frac{15}{20}$ = 20.75, but the short interaxial distance of 6.58 meters would calculate to exactly 11.20 braccio, making the interaxial proportion precisely $20\frac{15}{20}$ by $11\frac{4}{20}$ (simplified as $20\frac{3}{4}$ by $11\frac{1}{5}$), for $\frac{6.58\ m}{587\ mm} = 11\frac{4}{20}$ = 11.20. [38] What's important to keep in mind here is that Herzner's actual surveyed interaxial distance between columns never changes, only the hypothesized length of the braccia changes i.e. was a braccio of 586 or 587 millimeters used? It seems to me that a 'rational' measure of $20\frac{15}{20}$ by $11\frac{4}{20}$ braccio (according to a 587 mm braccio) is much more tenable than an 'irrational' measure of $20\frac{39}{50}$ by $11\frac{57}{250}$ braccia (according to a 586 mm braccio). There is evidence that Brunelleschi -- or a contemporary inspired by his revolutionary design of the Old Sacristy -- soon after the completion of San Lorenzo, also employed the same 587 mm braccio in the design of the Pazzi Chapel. It is well known that the architect of the Pazzi Chapel was not only directly inspired by Brunelleschi's design of the Old Sacristy at San Lorenzo, but, as evidenced through the astronomical dome over the altar, the architect (if not Brunelleschi himself) directly copied Brunelleschi's dome design of the Old Sacristy.[39] The similarities are so evident that an anonymous writer in 1490 credited Brunelleschi with the original design of the Pazzi Chapel. Through an on-site survey performed in 1976, Hecht concluded that a braccio of 587.5 mm was used in the Pazzi Chapel,[40] which, *being only a half of a millimeter longer than the 587 mm braccio*, leaves human error (either in construction or survey) a definite possibility. Indeed there

(2008). How Much Brunelleschi? A Late Medieval Proportional System in the Basilica of San Lorenzo in Florence. *Journal of the Society of Architectural Historians*, *67*(1), 18–57; ritter von Stegmann, C. M., Geymüller, H. v., Freiherr von Geymüller, H. A. (1924). The Architecture of the Renaissance in Tuscany, Illustrating the Most Important Churches, Palaces, Villas and Monuments. United States: Architectural Book Publishing Company.

[38] Clarke, Weights, Measures and Money or all Nations, p. 76. Clarke indicates that the braccia in use in Florence and Tuscany was divided into 20 soldi, therefore it makes sense that Brunelleschi's measure followed this division.

[39] Paul Barolsky. (2017). The Visionary Architecture of the Pazzi Chapel. *Arion: A Journal of Humanities and the Classics*, *25*(2), 1–10.

[40] Naredi-Tainer, 'Architektur und Harmonie', p. 117; Tavernor, 'Concinnitas in the Atchitectural Theory and Practice of Leon Battista Alberti', p. 294.

is no doubt that the Pazzi Chapel, Portinari's Chapel and the Tribune of Santa Maria delle Grazie were all inspired by Brunelleschi's design of the Medici's Old Sacristy at San Lorenzo (Figure 7).

In the early 1460s, Milan witnessed the introduction of Brunelleschi's architectural influence, particularly through the adoption of the 587 mm braccio at Santa Maria delle Grazie (SMG).[41] This unit of measurement, thought to be utilized by Brunelleschi in his designs for San Lorenzo and the Pazzi Chapel, creates an intriguing connection with Leonardo da Vinci's work in Milan. Notably, it correlates with the dimensions of the refectory wall Leonardo was tasked to decorate, which measured exactly 15 braccia, coinciding with the unit used for casting the bronze Sforza horse, as documented by Pacioli.

This parallel leads to thought-provoking speculation about Leonardo's selection of measurement units. Having been trained as a sculptor, painter and engineer in Florence by Verrocchio, a known associate of the Medici, Leonardo was certainly acquainted with the 587 mm braccio prevalent at San Lorenzo.[42] This raises an almost rhetorical question: Was Leonardo's use of the

[41] I have not yet done an analysis of other structures in Milan or Florence to see if the 587 mm braccia was in use. Despite my best attempt thus far, I cannot find any surveys or scaled drawings of the Portinari chapel, though I'm highly interested to see which braccia was in use in its design.

[42] A substantial part of Leonardo's recruitment to the Sforza court in 1483 was likely his ability to design and cast a magnificent bronze horse to commemorate the life of Ludovico's father and militant 'founder' of the Milanese duchy, Francesco Sforza (a Tuscan, born close to Florence). It is hardly a coincidence that at the same time Leonardo's mentor, Verrocchio, was awarded the contract to design and cast a magnificent bronze horse to commemorate the life of Bartolomeo Colleoni, the life-long Captain-General of Venice and Francesco Sforza's ally. Therefore, we cannot rule out that Leonardo was given the opportunity to build the Sforza horse primarily because Ludovico was aware that Leonardo was trained by Verrocchio and Verrocchio was awarded the contract in Venice. Through both Verrocchio and Leonardo's father Ser Piero da Vinci, it is commonly accepted that Leonardo was, at a minimum, on the border of the Medici's circle, and, at a maximum, a familiar with Lorenzo's gardens at San Marco and his personal ambassador, with Bernardo Rucellai, to Milan. Indeed, some of the important missions and commissions that he received, e.g. the Signori's Altarpiece for the Chapel of St. Bernard and the ambassadorial journeys to Milan, seem to indicate that he had direct artistic connections with Lorenzo and according to the Anonimo Magliabechiano, Leonardo was admitted by Lorenzo himself to study and work in the Medici gardens at the Dominican convent of San Marco. There is also evidence that in May, 1480 'Maestro Lionardo Dipintore' worked on the Tabernacle of the Sacrament located at San Lorenzo in the Medici sponsored 'Cappella del Corpo Cristo' adjacent to Brunelleschi's Old Sacristy.[42] It was a small but important commission which surely would have acquainted Leonardo with Brunelleschi's various devices. See Saalman, Filippo Brunelleschi: The Buildings p. 167, 437; Butterfield & Elam, 'Desiderio da Settignano's Tabernacle of the Sacrament', p. 335-340 Therefore, we're on good ground to think that the 587 mm braccia used by Leonardo to cast the Sforza horse was likely imported from Florence and used at Santa Maria delle Grazie before Leonardo's arrival to Milan in 1482. That Leonardo used this braccia in his design of The Last Supper is inferred not only by the preexisting dimensions of the Cloister of the Dead, Tribune and refectory which are precisely 60 : 30 : 15 braccia wide, respectively, but also by a clue that Pacioli left when he deliberately cut Leonardo's line AB into a perfect √5 : 1 ratio as I'll demonstrate in my forthcoming paper.

587 mm braccio for casting the bronze horse, as mentioned in Pacioli's "De Divina Proportione," a deliberate choice influenced by its use at SMG and his familiarity with it from his time in Florence, or was it simply a happy coincidence?

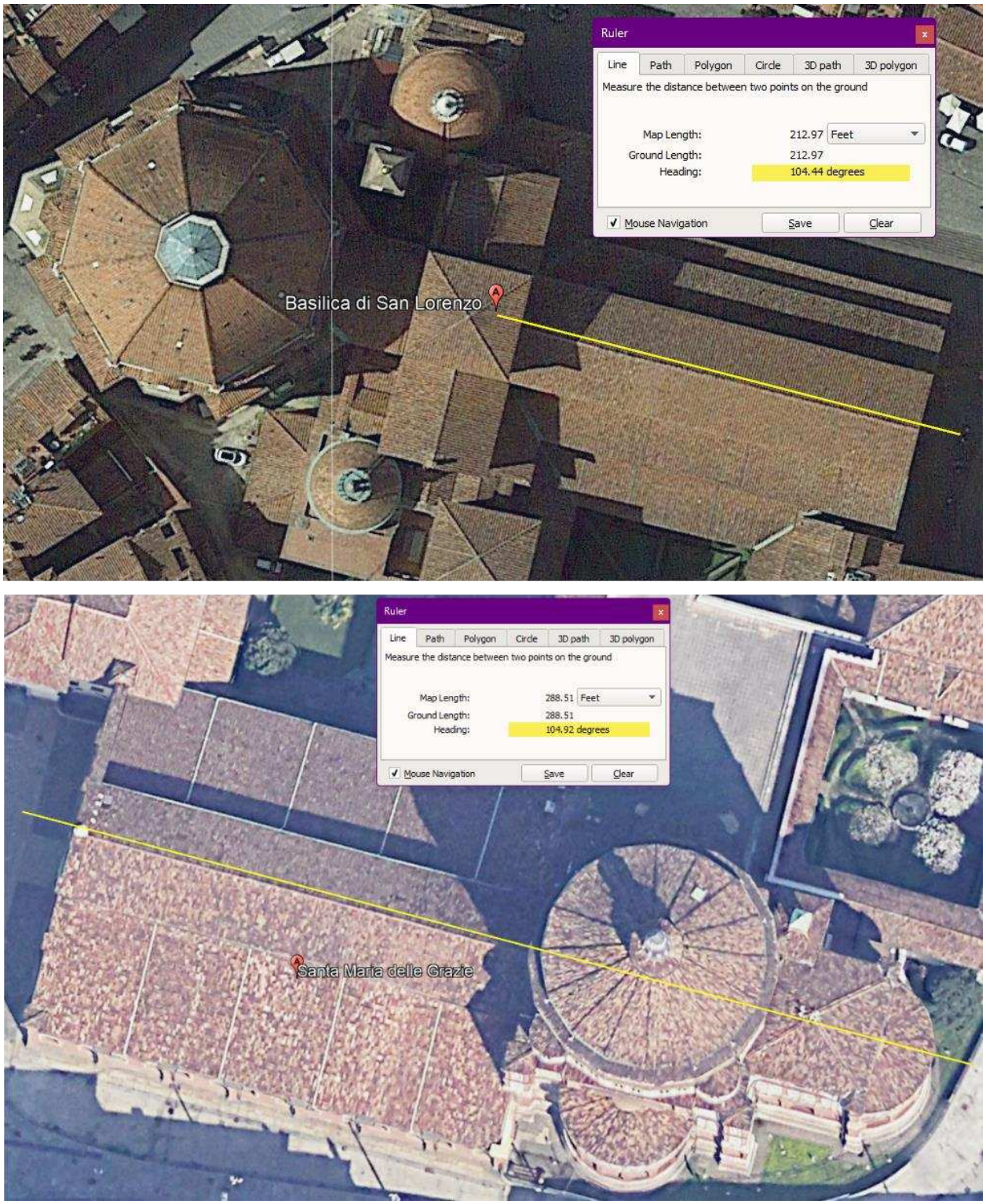


Figure 5 – The East-West liturgical axis of San Lorenzo (top) and SMG (bottom).

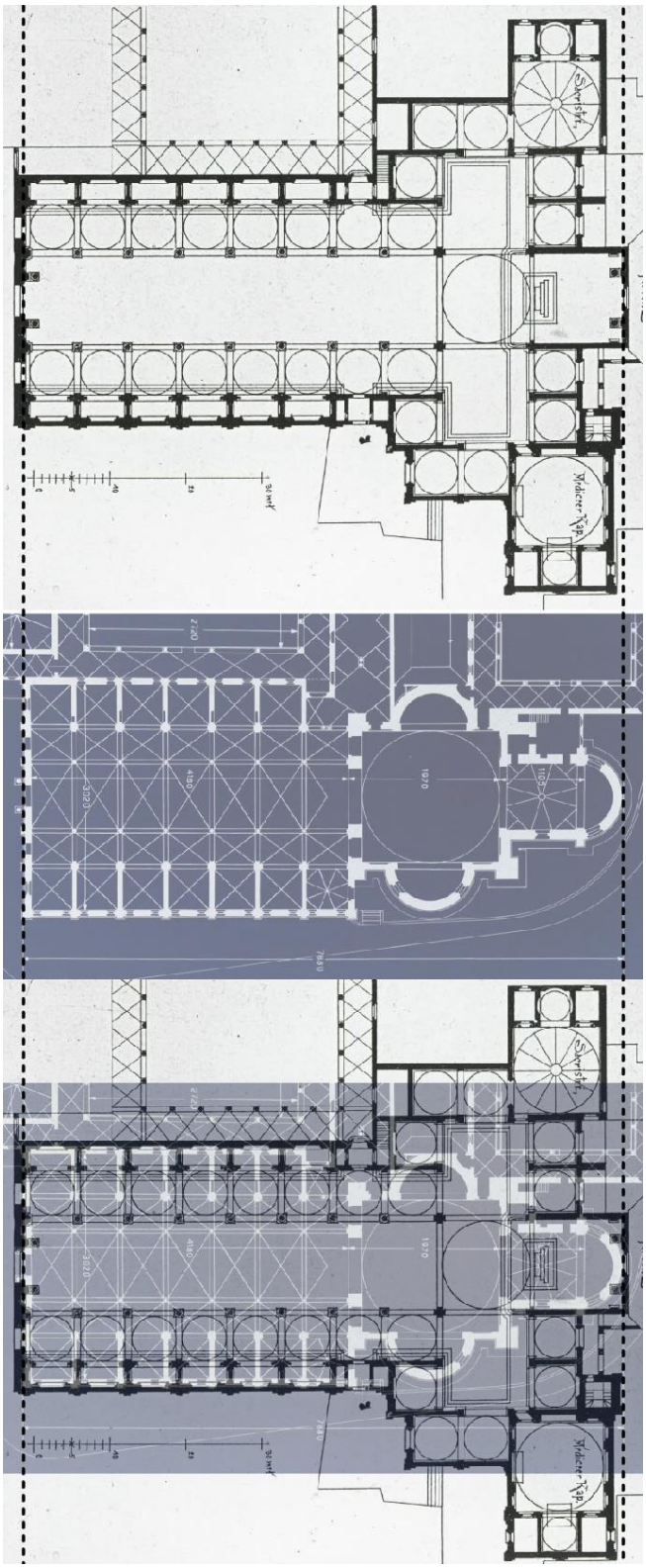


Figure 6 – Plan of San Lorenzo (top), SMG (middle) and combined (bottom).



Figure 7 – Brunelleschi inspired designs in Florence and Milan

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