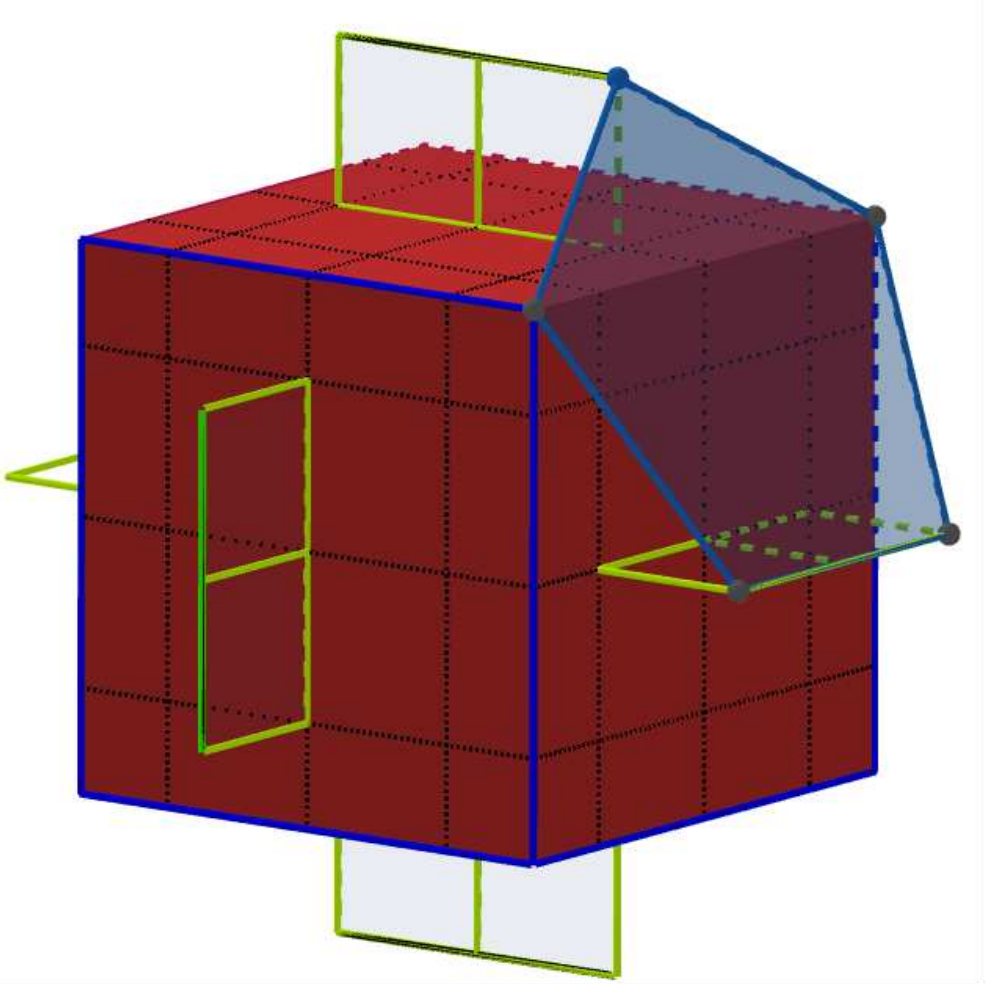


Euclidean Metrology and the Egyptian Duat

THE DODECAHEDRON AS THE BASIS OF ANCIENT COSMOLOGY & LARGE SCALE METROLOGY (LSM)

Carl Balowski | April 2024

Abstract

This study explores and builds on my hypothesis that ancient civilizations, particularly the Egyptians and the builders of Stonehenge, might have had an advanced understanding of celestial mechanics and Earth's dimensions, predating known measurement units like the royal cubit and megalithic yard with a measure akin to the modern meter. Through monumental constructions and precise astronomical observations, these ancients may have discerned the Earth's circumference, embedding this knowledge in structures like the Great Pyramid and Stonehenge. The text suggests these insights align with modern scientific understanding, highlighting Euclid's geometric construction of the dodecahedron from a cube via the golden ratio as a significant legacy of ancient knowledge. This geometric symbol, representing the Earth and the cosmos, was deeply infused with mathematical and metaphysical meaning, suggesting a sophisticated blend of science, philosophy, and spirituality in ancient metrological systems. The discussion extends to the Duat in Egyptian texts, portrayed through symbols like the encircled pentagon, reflecting a complex cosmology that integrates celestial and terrestrial realms. This exploration posits that ancient and contemporary measurement systems might share a profound connection, inviting a reevaluation of ancient societies' engagement with the universe through architectural and metrological strategies. This inquiry connects past architectural achievements and celestial engagements with current scientific discoveries, underlining humanity's enduring quest to understand the cosmic order.

Introduction

Venture beyond the confines of strict empiricism to consider an intriguing theory: the ancient Egyptians and the builders of Britain's Stonehenge might have possessed an advanced grasp of celestial mechanics and the Earth's dimensions, potentially predating the invention of the royal cubit and the megalithic yard with a unit of measure similar to the modern meter. Esteemed for their monumental constructions and meticulous astronomical observations, these ancient peoples could have either inherited or unveiled the Earth's circumference through their intensive study of the cosmos and mathematics. Subsequently, they may have encoded this profound understanding into the fabric of the Earth itself, through the erection of structures like the Great Pyramid and Stonehenge.

The ancient world's deep engagement with celestial wonders, potentially merged with now-lost methodologies, suggests a level of insight comparable to that of modern science, spanning millennia in a unified journey of discovery. Within this venerable legacy, Euclid's (a Greek-Egyptian) graceful construction of the dodecahedron from a cube, through the application of the golden ratio, stands as a significant vestige of such ancient knowledge. To those early observers, the cube symbolized Earth, and the dodecahedron represented the celestial realm—specifically, for the Egyptians, the Duat. Therefore, this geometric relationship between the Cube and Dodecahedron was not merely symbolic but was imbued with significant mathematical and metaphysical implications, resonating deeply with the era's geometers and cosmologists, and potentially the High Priests of Egypt. The encapsulation of the cube by the dodecahedron, representing the Earth ensconced within the vast celestial expanse, hints at a divine architectural blueprint underlying ancient metrology systems. Such imagery suggests that the ancients, presuming to be guided by a divine intellect, crafted these forms as foundational archetypes for metrology, directly influencing construction, land measurement, and the erection of edifices to honor the divine.

We can be confident that Euclid's method for constructing the dodecahedron atop a cube, deeply intertwined with the golden ratio, surely did not emerge in isolation. By examining the simple geometric principles behind Euclid's design, we might reveal intriguing parallels between the metrological frameworks of ancient civilizations and contemporary systems. Such parallels could demonstrate a significant link between measurements of the Earth and the cosmos, hinting that a doppelgänger to the modern meter may have influenced the metrological insights of ancient Egypt. The idea that the dodecahedron's design—rooted in both celestial and terrestrial measurements—could have been pivotal in shaping ancient measurement systems prompts a reevaluation of the ways ancient societies engaged with the cosmos and developed their architectural and metrological strategies. This inquiry not only highlights the architectural splendor and astral engagements of the ancients but

also connects these endeavors to modern scientific discoveries, illustrating an enduring pursuit of knowledge and a deep respect for the cosmic order that binds the universe.

The Symbolism of Determinatives in Egyptian Texts

In exploring the multifaceted interpretations of the Duat as depicted in ancient Egyptian funerary texts, it becomes apparent that divergent perspectives arise depending on the focal point of the respective authors—whether celestial or terrestrial in nature. The term "Duat" itself unfolds a tapestry of meanings, a complexity further unraveled through the lens of cognitive linguistics, particularly through the meticulous analysis of determinatives found within Egyptian texts. These determinatives transcend mere linguistic adornments; rather, they function as potent icons, encapsulating profound cultural insights and disclosing intricate symbolic networks. Consequently, delving into the significance of these determinatives affords a profound glimpse into the cognitive landscape of ancient Egyptian thought, tracing the evolutionary trajectory of afterlife concepts as evidenced within the Pyramid and Coffin Texts.

Depicting the Duat in Pyramid Texts

Within the ancient Egyptian Pyramid Texts, the concept of the Duat is frequently represented by a five-pointed star enclosed within a circle, though it is sometimes simply depicted as a standalone five-pointed star (Figure 1). These oldest of texts, unlike the texts of the new kingdom, most often describe the Duat not as an 'underworld' but as a celestial entity that encircles the cosmos.[1] Out of 81 references to the Duat found within these texts, 55 depict it with a five-pointed star encased in a circle, 16 feature a five-pointed star without any encirclement, and the remaining 10 predominantly illustrate it with a 45° diagonal-cross within a circle. These rare determinatives, such as a cross within a circle, deepen our understanding of the Duat, suggesting an interplay between the 5-pointed star and the four cardinal directions of the square-like, earthly domain. These symbols in textual context coalesce to furnish a nuanced understanding of the Duat's cosmological significance not unlike the Platonic conception of the dodecahedron (the fifth element), which as a celestial canopy, surrounds the cube like earth. Central to this conception was the veneration of the northern sky's axis, intertwined with the mythos of the Undying Stars. Within this framework, sovereigns and gods were envisaged as eternal denizens travelling 'through' the Duat (consider the constellation Orion), actively participating in the cosmic ballet that allegorized unending cycles of rejuvenation, mirroring the eternal continuum of existence beyond mortal demise.

[1] Zago, S. (2018). Imagining the Beyond: The Conceptualization of Duat between the Old and the Middle Kingdoms. *Journal of the American Research Center in Egypt*, *54*, 203–217.

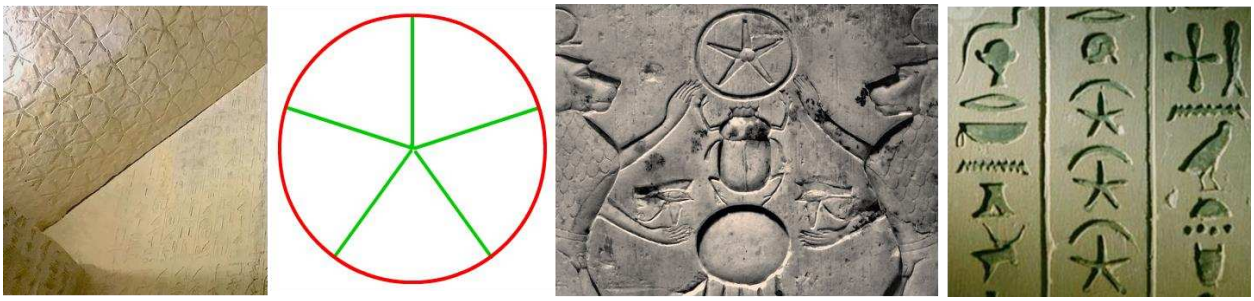


Figure 1

The Encircled Five-Pointed Star: A Cosmic Emblem

The encircled five-pointed star, as elucidated within the ancient Egyptian textual corpus and specifically in relation to the Duat, emerges as an emblematic representation of a comprehensive cosmic realm. This imagery serves to portray the Duat as an expansive domain encompassing celestial spheres and the perpetuity of rebirth, accentuated by the auspicious phenomenon of solstices and heliacal star risings. Implicit within this visual motif is the notion of the Duat as a liminal space, bridging the earthly and celestial planes, thereby embodying both a tangible and metaphysical locus wherein the cyclical rhythms of day and night, visibility and obscurity, mirror the eternal odyssey of the soul in tandem with celestial entities. Therefore, the pentacle—a five-pointed star enclosed within a circle—emerges as a powerful symbol, signifying the Duat's function as a bridge linking the earthly and the heavenly spheres, embodying the deep cosmological understanding and spiritual ambitions of the ancient Egyptian civilization.

Complexity in Simplicity: The Encircled Pentagon

This observation prompts the question: Why would the Egyptians choose to represent the Duat with an encircled five-pointed star, highly indicative of the pentagon, rather than with a simpler-to-draw or carve encircled triangle or square? Drawing triangles and squares is generally easier than drawing a pentagon due to their simpler geometric properties and compatibility with standard drawing tools. Triangles and squares have fewer sides, reducing the complexity of measurements and construction. Squares have uniform 90° or 45° angles and equal side lengths, and equilateral triangles have 60° or 30° equal angles and sides, making them straightforward to construct with rulers and compasses or ropes. Conversely, pentagons and pentagrams require precise internal angles of 36°, 72° or 108° with equal side lengths, involving more complex geometric constructions, such as dividing a circle into five equal parts or utilizing the golden ratio, which adds a layer of complexity. The simplicity of constructing triangles and squares stems from their straightforward geometric

ratios and the ability to easily bisect angles and create equal segments, unlike the cosmic process needed for a regular pentagon as articulated by Plato. Therefore, the choice of the Egyptians to use an encircled pentagon as a symbol for the Duat (cosmos) was not rooted in simplicity but in its profound geometric and perhaps cosmological implications. The symbol of an encircled pentagon not only combines the geometric perfection of the circle, expressed through the irrational number pi, but also embodies the pentagon, intimately associated with the golden ratio and the dodecahedron, which Plato also claimed that God used as the fabric of the 'circular' celestial realm. This choice underscores the Egyptians' deep understanding of geometry and mathematics, choosing a symbol that encapsulates the cosmic significance and inherent harmony of the universe, as reflected in the interplay of pi and the golden ratio within the encircled pentagon.

Euclid's Elements and Ancient Mathematics

In the esteemed annals of mathematical history, 'Euclid's Elements' holds a place of unparalleled reverence. From its inception in ancient Alexandria to its enduring relevance in the modern era, the Elements, working from first principles to the construction of the dodecahedron, has served as a cornerstone of mathematical education, fostering a rigorous approach to geometric reasoning and deductive logic that laid the groundwork for the development of modern mathematics.

To appreciate the genesis of Euclid's magnum opus, one must delve into the life and milieu of its esteemed author. Euclid, believed to have lived ca 300 BC, flourished in the vibrant intellectual crucible of Alexandria, just in its infancy. Situated at the crossroads of diverse cultures, Alexandria catalyzed a fertile environment conducive to intellectual ferment and innovation. As a Greek-Egyptian, Euclid likely drew upon a rich tapestry of mathematical and spiritual wisdom inherited from illustrious predecessors such as Pythagoras and Theaetetus, whose seminal contributions laid the groundwork for the geometric principles elucidated in the Elements.

However, as a denizen of Alexandria, Euclid would have been immersed in the massive tradition and corpus contained in the recently founded Library of Alexandria—a repository of knowledge gathering texts from various ancient civilizations, including the Egyptians and Babylonians. Within this cultural milieu, Euclid undertook his monumental endeavor to systematize and codify the principles of geometry into a coherent and comprehensive treatise.

Exploring the Foundations of Euclid's Elements

At the core of Euclid's Elements lies a comprehensive square and compass exploration of geometric propositions and demonstrations, meticulously formulated with exacting logical rigor and axiomatic precision. The treatise unfolds across thirteen meticulously organized books, each devoted to a specific aspect of geometry. Beginning

with the foundational principles of plane geometry, the text progresses towards the intricate properties characterizing three-dimensional solids. Throughout this structured journey, Euclid's overarching objective is the construction and contemplation of the Platonic Solids, culminating in the pinnacle of geometric perfection: the Dodecahedron. With its twelve pentagonal faces, the Dodecahedron represents the epitome of Euclid's geometric inquiry and the embodiment of timeless mathematical elegance.

The Role of Geometric Algebra in Ancient Surveying and Planning

The foundational Proposition 1 of Book I provides a systematic method for constructing an equilateral triangle—a fundamental building block upon which subsequent geometric reasoning is predicated. Book Two emerges as a seminal exploration into the realm of geometric algebra. This profound foray into the intricacies of geometric areas—particularly through the examination of squares and rectangles—unveils not only a theoretical marvel but also a treasure trove of practical applications. These principles, meticulously laid out across Euclid's propositions, are of monumental importance, serving as the bedrock upon which ancient surveyors, planners, and architects could erect the edifices of civilization itself.

Book Two, with its focus on the algebraic treatment of geometric figures, presents a methodology for equating and transforming the areas of geometric shapes. This is epitomized in the practice of determining the area of a square constructed upon the diagonal of a rectangle, an endeavor that inherently relates the sum of areas of squares on the rectangle's sides to the area of the square on its diagonal. Such a proposition not only underpins the Pythagorean theorem but also becomes a pivotal technique in the disciplines of surveying and architectural planning. The mathematical underpinnings provided here are crucial for accurately defining property boundaries, devising urban layouts, and the equitable allocation of land, thereby facilitating the management of agricultural expanses and the efficient design of civic spaces.

For the practitioners of ancient civilizations—surveyors, architects, and planners—the geometrical principles elucidated in Book Two of the *Elements* were indispensable. These principles empowered surveyors with the means to measure land with unprecedented accuracy, enabling the precise demarcation of lands and the judicious distribution of resources. Architects and planners, leveraging Euclid's insights, could embark on designing structures and spaces that were not only aesthetically pleasing and functionally apt but also resonant with the mathematical harmony of stability and proportionality. The abstract algebraic treatment of geometric figures laid out by Euclid thus opened avenues for sophisticated architectural designs, where the harmonious partitioning of space and the aesthetic amalgamation of geometric forms could be achieved with sublime finesse.

The Golden Ratio: Bridging Geometry with Cosmic Harmony

Of all the propositions in Euclid's Book Two, number 11 (Euclid 2:11) emerges as a pinnacle of his geometric and algebraic mastery. It is so significant, that the same formulation, applied to a square, is replicated in Book 13, Proposition 4 (Euclid 13:4) as a precursor to the construction of the Dodecahedron. This proposition illustrates the method for creating a rectangle with an area identical to that of a specific square, by dividing a line segment at the point known as the "golden ratio." This strategic division highlights Euclid's profound understanding of proportion and balance, marking a significant integration point within the Elements. The golden ratio, celebrated for its beauty and mathematical importance in the natural sciences and represented today by the Greek letter phi (Φ), is strikingly illustrated in this proposition. It underpins not only the proportions of the inscribed pentagon as demonstrated in the ‘Duat’ constructions of Euclid 4:10-14, but also serves as a foundational principle for the whole of Book 13 and the dodecahedron, epitomizing the ‘quintessence’ of Euclid's geometrical exploration.

The Dodecahedron: A Symbol of Universal Harmony and Mathematical Rigor (Euclid to Pacioli)

At the heart of the dodecahedron's geometric marvel lies the core principle of the golden ratio as demonstrated in proposition 13:5. As the epitome of the Platonic solids, the dodecahedron embodies the universe's harmony as viewed through the lens of mathematical irrationality, and is increasingly being found to play a crucial role in the construction of ancient holy sites. These structures, as seeming precursors to Euclidean geometry, were conceived as earthly mirrors of the celestial harmony, creating a tangible connection between the divine and the mundane realms.

Euclid's vision skillfully integrates the intellectual traditions of Greece and Egypt, presenting a model of the cosmos that marries mathematical rigor with spiritual depth. This intersection of mathematical precision and mystical insight is much later explored in Luca Pacioli's seminal work, "De Divina Proportione." Within its pages, the sacred essence of the dodecahedron is examined with profound Christian reverence, elevating this Platonic solid to a primary focus of theological and artistic investigation. Deeply influenced by Euclidean geometry, Pacioli's treatment of the dodecahedron is imbued with a sense of sanctity reminiscent of the ancient Egyptians' reverence for the Duat, perceived as the very fabric of the Cosmos. Through his narrative, the dodecahedron is elevated to a status akin to ancient, holy symbols, revealing Pacioli's deep admiration for the geometric form as a conduit to the divine, reflecting the philosophical and spiritual inquiries that have captivated humanity across different epochs. In this way, the dodecahedron, embellished with twelve pentagonal faces, transcends mere physicality to symbolize a higher order, intricately tied to the enigmatic golden ratio.

Pacioli’s integration of arithmetic with Euclidean principles, particularly highlighted in his ‘exegesis’ Euclid’s proposition, lays the foundational understanding of the dodecahedron and its fellow Platonic solids. These solids are now defined by their uniformity, convexity, symmetry, and strict adherence to Euler’s formula, setting them apart in the realm of three-dimensional geometry. Their aesthetic appeal and mathematical elegance arise from these defining features, which emphasize the geometric exclusivity and cosmic uniqueness of these five forms.

Plato's Cosmic Correlation with the Platonic Solids

In Plato's "Timaeus," the Platonic solids are meticulously correlated with the elements of the cosmos: fire is represented by the tetrahedron, earth by the cube, air by the octahedron, water by the icosahedron, and the heavens by the dodecahedron. This correlation reflects the unique characteristics of each solid, such as the tetrahedron's pointed sharpness—constructed from four equilateral triangles—embodying the penetrating nature of fire, and the cube's robust stability—being the only Platonic solid formed from squares (which themselves derive from two right-angled isosceles triangles)—mirroring the solidity of earth. Among these, the dodecahedron, with its intricate structure based upon the golden ratio, stands apart as a geometric marvel, suggesting a divine or celestial superiority. The construction of these solids fundamentally relies on triangles, highlighting Plato's conviction, influenced by Egyptian and Pythagorean thought, that the triangle serves as the foundational building block of the universe.

The Golden Ratio and the Dodecahedron's Divine Geometry

The dodecahedron, unlike the other Platonic solids, which are constructed from the ‘standard’ right-angled or equilateral triangles, is based upon an isosceles triangle that adheres to the golden ratio. This specific triangle, with sides in the golden ratio, diverges from the simpler geometric principles that govern the construction of its counterparts. The golden ratio, a principle that has captivated mathematicians and artists alike, imbues the dodecahedron with a divine and mysterious status, aligning it with Plato's view of it as composing the framework of the celestial sphere.

This distinction underscores why Plato attributed a celestial and divine nature to the dodecahedron, elevating it above the other solids in symbolic significance. The use of the golden ratio in its construction not only adds a layer of mathematical complexity but also a depth of philosophical and cosmic meaning. Plato's conception of the dodecahedron as representative of the celestial sphere suggests a universe not merely structured by geometric order but also imbued with a profound, underlying enigma that mirrors the golden ratio's ability to self-replicate.

Euclid's Contribution to the Celestial Framework and Spiritual Exploration

Euclid's "Elements" further explores this enigmatic connection, with the culmination of its geometric explorations leading to the construction of the dodecahedron, symbolizing the pinnacle of geometric reasoning and insight. Its construction 'on top of' a cube, as demonstrated by Euclid, resonates with Plato's and ancient Egyptian's symbolic representations of the earth encased by the celestial, a theme recurrent in the conception of the cube and the dodecahedron, as well as the Egyptian Duat encircling the earth.

In the historical narrative of the Platonic solids, we witness a continuum of intellectual and spiritual exploration that spans the history of thought. From the hands of prehistoric Albion sculptors (Figure 2) to the philosophical and mathematical elucidations by Plato, Euclid, and Pacioli, this journey touches the essence of reality, showcasing humanity's ceaseless quest to decipher the cosmos through the lens of geometry.[2] Thus, the dodecahedron's distinction, rooted in its construction of a double-square atop a cube, elevates it to a model of cosmic unity and divine mystery between heaven and earth. It embodies the celestial order governing the cosmos, inviting those stuck on the ‘four corners of the earth’ to look up and ponder the intricate fabric of existence. Through this geometric form, the ancients envisioned a cosmos that was divinely ordered, reflecting a deep interconnection between mathematics, philosophy, and the spiritual, governed by the elegant and sometimes enigmatic principles of geometry.



Figure 2

[2] Emulation of the Platonic Solids were found among the ancient stone balls at Ashmolean and are estimated to be as old as Stonehenge.

The Double-Square: Linking Ancient Egyptian Architecture and Cosmic Order

In my study on the Giza Grid, it's evident that the concept of the double-square and the square root of 5 held significant importance for ancient Egyptian architects. This is illustrated through the design of the King's Chamber, Hemiunu's mastaba, and the combined diagonal of all three Giza Pyramids, underscoring their relevance. Therefore, the prominent role of the double square in the construction of the dodecahedron atop a cube aligns perfectly with these architectural instances, highlighting a continuity with primoridal geometric principles rather than presenting an anomaly.

The dodecahedron's legacy, immortalized in stone and thought, forges a bridge that connects ancient wisdom with today's scientific discoveries,[3] therefore the dodecahedron ascends beyond its geometric confines to reflect the cosmos's mystical structure, embodying a microcosm of the universe itself akin to the Egyptian Duat, the encircled pentagram and/or square. As contemporary science explores the intriguing possibility that the universe's shape might mirror that of the dodecahedron, this venerable Platonic solid captivates the imagination anew. It serves not only as a gateway to the mathematical foundations of existence but also as a profound metaphor for the cosmic order orchestrating all of creation.

Visual Exploration of Euclid's Dodecahedron and Its Metrological Implications

Now, on the cusp of further discovery, we are poised to embark on a visual exploration of Euclid's dodecahedron (Figure 3). This forthcoming phase of our journey aims to dissect and examine the foundational geometry underpinning its structure visually, seeking to illuminate how this geometrical 'ideal' may sit as the bedrock of early Egyptian metrology. This endeavor promises not only to deepen our understanding of the dodecahedron's geometric intricacies but also to enhance our appreciation of its role as a timeless emblem of cosmic symmetry and harmony.

[3] Pellis, Stergios, Poincaré Dodecahedral Space Solution of The Shape of The Universe (December 31, 2022).

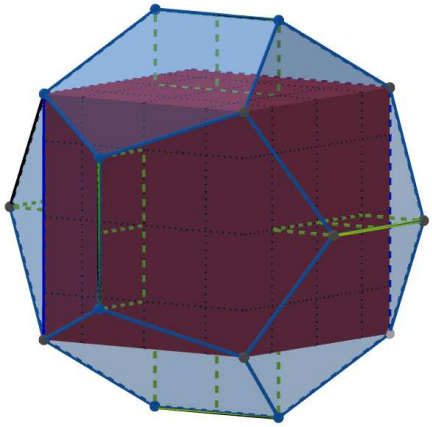


Figure 3

The Dodecahedron's Construction

On initial observation, creating a dodecahedron using just a square and compass might appear to be a daunting challenge. Yet, I will show that this task only requires an understanding of two fundamental geometric principles, knowledge that was undoubtedly accessible to the ancient Egyptians as well as Euclid's predecessors. Armed with these principles, one simply needs a dash of straightforward ingenuity to piece everything together. The first principle involves constructing a cube, while the second entails dividing a square according to the golden ratio. Later I'll reveal the secret sauce that unlocks its simplicity.

Step 01 (Figure 4):

Construct a cube and then quarter it. Notice that each cube face is now composed of four 'quarter' squares.

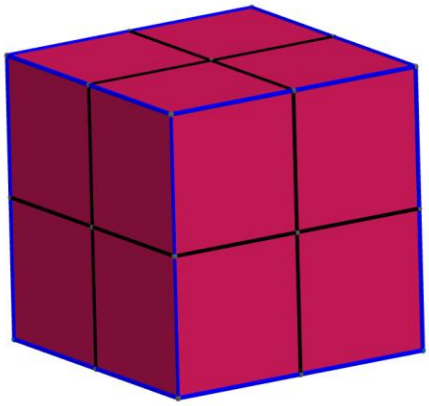


Figure 4

Step 02 (Figure 5):

According to Euclid 13:4, segment each quarter-square into golden ratio squares.

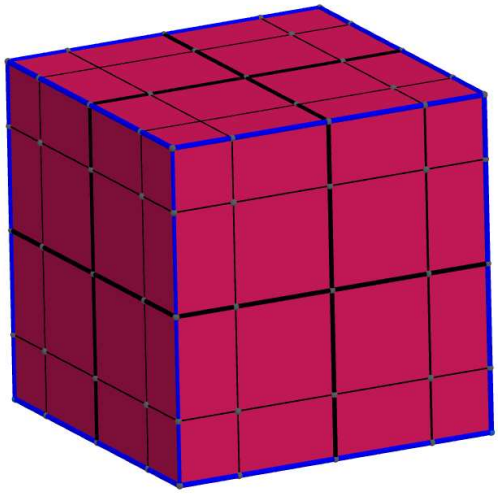


Figure 5

Step 03 (Figure 6):

From the center of each cube face, raise each double-square perpendicularly.

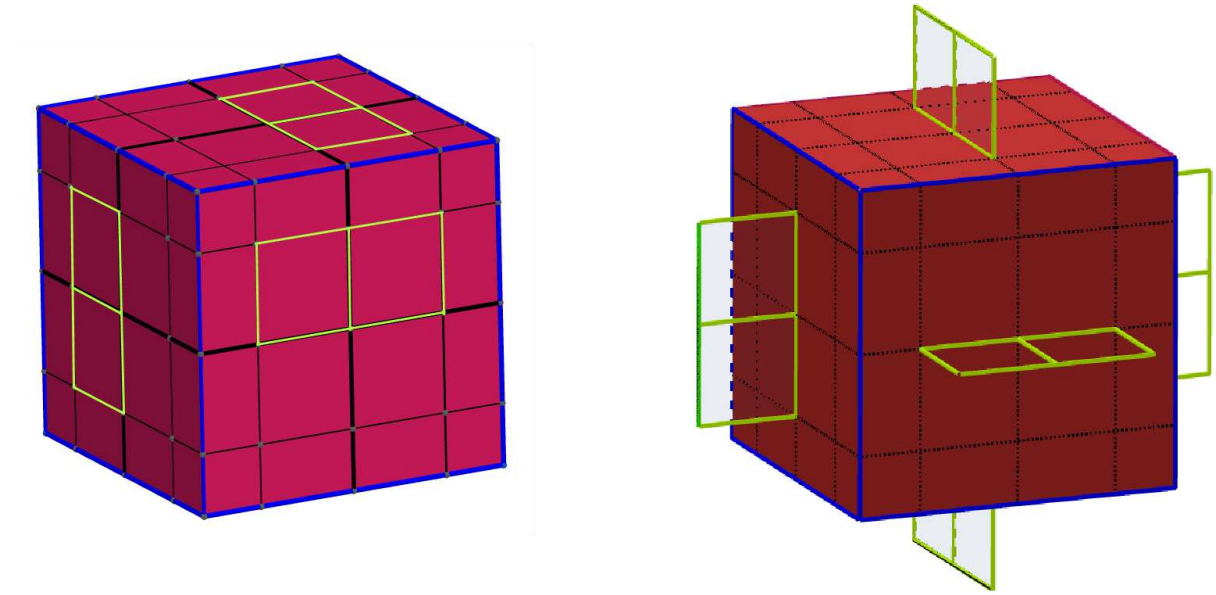


Figure 6

Step 04 (Figure 7):

Connect each vertex of the cube to the closest corner of the double-square.

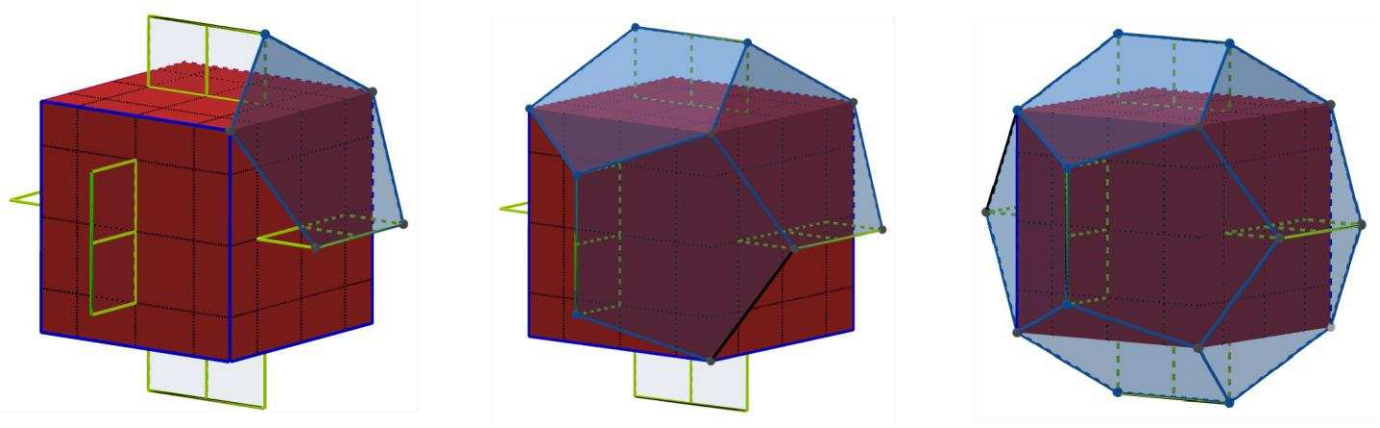


Figure 7

And there you have it, with straightforward steps, we've assembled twelve pentagonal faces on top of a cube, a figure that Plato linked with the celestial canopy! This geometric endeavor carries profound metaphysical implications, resonating with the age-old view of the cube as an earthly symbol surrounded by the dodecahedron as the celestial sphere. Thus, a clever glyph for the dodecahedron could be the pentagram within a circle, esteemed in ancient Egypt as the emblem of the Duat, the celestial realm which encircles the earth. Therefore, a mathematical exploration of the dodecahedron's structure might just reveal important information about ancient metrics and their association with the earth and/or stars. To explore this intriguing hypothesis, we should take a closer look at the underlying geometric methodology.

Exploring the Golden Ratio in Dodecahedron Construction

The essence of the dodecahedron's construction relies on dividing a cube face four times according to the golden ratio. Euclid outlines a straightforward method for achieving this division in his work, particularly noted in Proposition 13:4 (Figure 8). He states,

"If a straight line is cut in extreme and mean ratio (the golden ratio), the squares on the entire line and the shorter segment together equal three times the square on the longer segment."

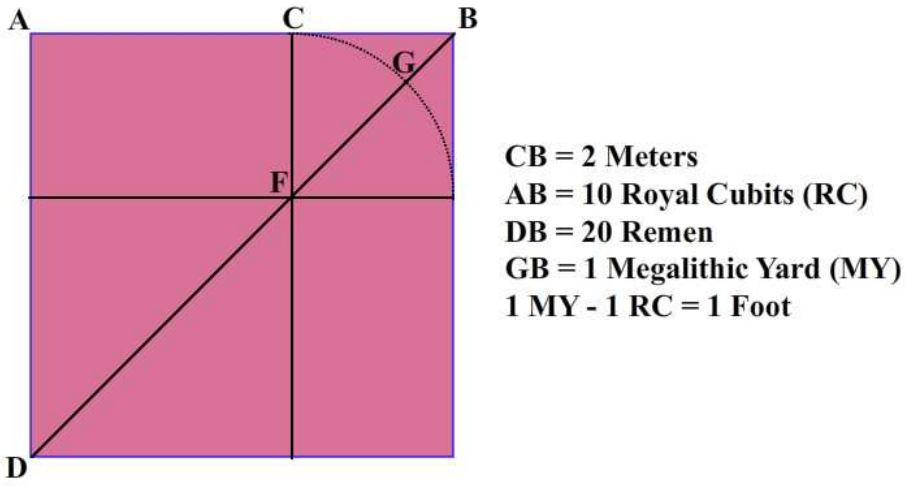


Figure 8

In the figure above, Euclid's exploration of the golden ratio within a square reveals a fascinating geometric relationship: by dividing a whole square using the golden ratio, the two resulting squares inside the original square are not only proportional to each other according to the golden ratio, but the larger of the two also shares this proportion with the whole square. While intriguing on its own, the real marvel lies in the arithmetic proof suggesting that the sum of the areas of the smaller square and the whole square is three times the area of the larger nested square alone, expressed as $AB^2 + CB^2 = 3AC^2$.

Ancient Egyptian Mathematics and the Base-10 System

The ancient Egyptians pioneered a non-positional base-10 numbering system. Their mathematics, evident in hieroglyphs, employed symbols for powers of ten to represent numbers, facilitating operations like addition, subtraction, multiplication, and division, primarily using unit fractions. This numeric system was integral to commerce, astronomy, surveying, and monumental constructions, showcasing their mathematical sophistication. Additionally, the Egyptian solar calendar, structured around a base-10 system and the 'decan' further underscores their influence on subsequent civilizations and the enduring legacy of base-10 in modern mathematics and science.

Therefore, the connection to our study becomes clear once we assign a length of 10 Royal Cubits to line AB. This results in the calculation of line AC as 6.18033 Royal Cubits, line CB as 3.81967 Royal Cubits, and line DB as 1.41421 Royal Cubits. Predictably, line DB corresponds to 20 Remen, but surprisingly, it also shows that the length of CB is exactly 2 meters! Therefore, with AB set to 10 Royal Cubits, a single Royal Cubit is equivalent to 0.5236067 meters. The intrigue intensifies when we examine that -- like the Remen's relation to the Royal Cubit -- GB merely represents the difference between CB and its diagonal FB. This precise relationship not only defines

the Megalithic Yard at 0.828427 meters but also highlights that the difference between 1 Megalithic Yard and 1 Royal Cubit is virtually identical to 1 Foot (99.993%)! In essence, what we are witnessing is the convergence of four measurement systems - Metric, Egyptian, Megalithic, and Imperial - within a singular, simple construction that has been overlooked for 2300 years, yet flawlessly symbolizes the cosmic construction as envisioned by the ancients.

This observation is intriguing, uncovering a simple yet profound geometric principle by Euclid – who was an Egyptian -- that serves as a foundation for constructing a dodecahedron using a cube. Furthermore, this mathematical principle can be easily demonstrated through straightforward geometric arithmetic, a technique undoubtedly familiar to ancient civilizations such as the Egyptians and Babylonians, as indicated by their writings on papyrus and clay tablets. This knowledge was further developed by subsequent mathematicians like Archimedes, Diophantus, Abul-Wafa, Leonardo of Pisa, and Luca Pacioli, marking a noteworthy distinction: while the Egyptians utilized a non-positional base-10 fractional system, later geometers adopted a base-10 positional system, as follows:

$$AB = 10\ Royal\ Cubits = 5.2360679\ meters$$

$$AC = (\sqrt{125} - 5) = 6.180339\ Royal\ Cubits = 3.2360679\ meters$$

$$CB = (15 - \sqrt{125}) = 3.81966\ Royal\ Cubits = 2\ meters$$

$$DB = (AB\ \sqrt{2}) = 14.14213\ Royal\ Cubits = 7.404918\ meters = 20\ Remen$$

$$GB = CB(\sqrt{2} - 1) = 1.582154\ Royal\ Cubits = 0.828427\ meters = 1\ Megalithic\ Yard\ (MY)$$

$$\frac{AB}{10} = 1\ Royal\ Cubit\ (RC) = 0.5236067\ meters$$

$$(1\ MY) - (1\ RC) = 0.30482\ meters = 1\ Foot\ (99.993\%)$$

Now, if we were to reduce the calculations by a factor of 10, so that AB was equivalent to 1 Royal Cubit, then the arithmetic would be the same, but take on a different appearance, like so:

$$AB = 1\ Royal\ Cubit = 0.52360679\ meter$$

$$AC = \frac{\sqrt{5}-1}{2} = 0.6180339\ Royal\ Cubits = 0.32360679\ meters$$

$$CB = \frac{3-\sqrt{5}}{2} = 0.381966\ Royal\ Cubits = 0.2\ meters$$

$$DB = (AB\ \sqrt{2}) = 1.414213\ Royal\ Cubits = 0.7404918\ meters = 2\ Remen$$

$$GB = CB(\sqrt{2}-1) = 0.1582154\ Royal\ Cubits = 0.0828427\ meters = 0.1\ Megalithic\ Yard\ (MY)$$

$$\frac{AB}{10} = 0.1\ Royal\ Cubit\ (RC) = 0.05236067\ meters$$

$$(0.1\ MY) - (0.1\ RC) = 0.030482\ meters = 0.1\ Foot\ (99.993\%)$$

Innovative Measurements and the Golden Ratio

Numerous methods exist for catching fish, and similarly, if the meter was recognized before the Royal Cubit or the Megalithic Yard, an intriguing alternative approach (referenced in Euclid's subsequent construction at 13:5) could begin with establishing line CB as either 1 or 2 meters. This line is then extended by the golden ratio to create line AB, measuring either 2.618033 or 5.236067 meters (equivalent to 5 or 10 Royal Cubits). By constructing squares along lines AB and CB, this alternate geometric configuration would equally lay the groundwork for the dodecahedron's construction, showcasing the golden ratio's integral role in aligning ancient measurements with the universal order.

We've previously explored how Proposition 13:4 from Euclid's "Elements" plays a crucial role in the construction of the dodecahedron and its connection to four different metrological systems. To gain a deeper understanding of the dodecahedron's symmetry and its relationship with these systems of measurement, it's beneficial to examine it through an orthogonal view.

Understanding the Dodecahedron Through Orthogonal Views

An orthogonal view of a Platonic solid such as the dodecahedron allows us to see its faces, edges, and vertices clearly, without the distortions of perspective (Figure 9). In this view, lines that might converge in three-dimensional space are shown as parallel, offering a clear and undistorted depiction of the solid's geometric properties. This perspective is invaluable for appreciating the uniform edge lengths and angles of the

dodecahedron, illustrating how its pentagonal faces come together seamlessly. It precisely demonstrates the solid's regularity and the harmonious tessellation of its faces.

This viewing method is especially insightful for understanding the dodecahedron's structure, as it lays out the relationships between adjacent faces and how they are arranged around each vertex. It presents a straightforward, undistorted view of the solid's symmetry, aiding in comprehending the dodecahedron's intricate shape and its symbolic significance among the Platonic solids as a representation of the cosmos.

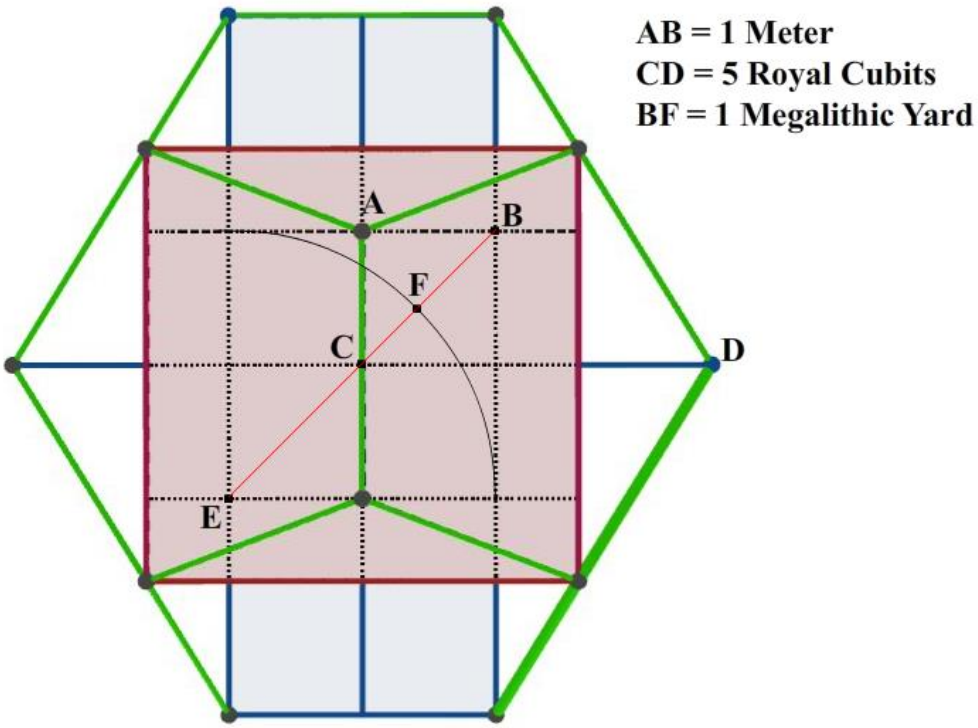


Figure 9

Exploring Metrological Correlations Through Geometric Projections

By adopting an 'edge-on' orthogonal perspective of the dodecahedron, we observe how it encapsulates a cube within its geometry, with the cube's face appearing as a square within the six-sided orthogonal projection of the dodecahedron. Setting line CD, the dodecahedron's mid-radius, to 5 Royal Cubits reveals that line AB measures 1 Meter, and line FB measures 1 Megalithic Yard. This observation opens up myriad possibilities for exploring the dodecahedron's construction and finding metrological correlations between systems. However, instead of

delving into further demonstrations, I'll end by presenting the basic pentagram, which itself might be the most simplistic way of demonstrating the correspondence between the Royal Cubit and meter.

Concluding Insights: The Pentagram and Metrological Correspondence

Readers interested in geometry who are now examining Euclid's methods and calculations are likely to quickly discern that the core relationship between the meter and the Royal Cubit derives from the squared golden ratio, or a 1 : 2.618033 ratio. This ratio is not just central to Euclid's designs but is fundamental to the geometry of both the pentagon and the dodecahedron. Consider the design of a pentagram, reminiscent of the Duat (Figure 10). Drawing a pentagram naturally forms both an inner and an outer pentagon. Assigning a meter's length to the inner pentagon's side, various lengths in the construction, including the outer pentagon's side, would precisely match 5 Royal Cubits. This simple yet profound symbol demonstrates the potential interconnectedness of ancient measurement systems, and the natural proportions dictated by the golden ratio.

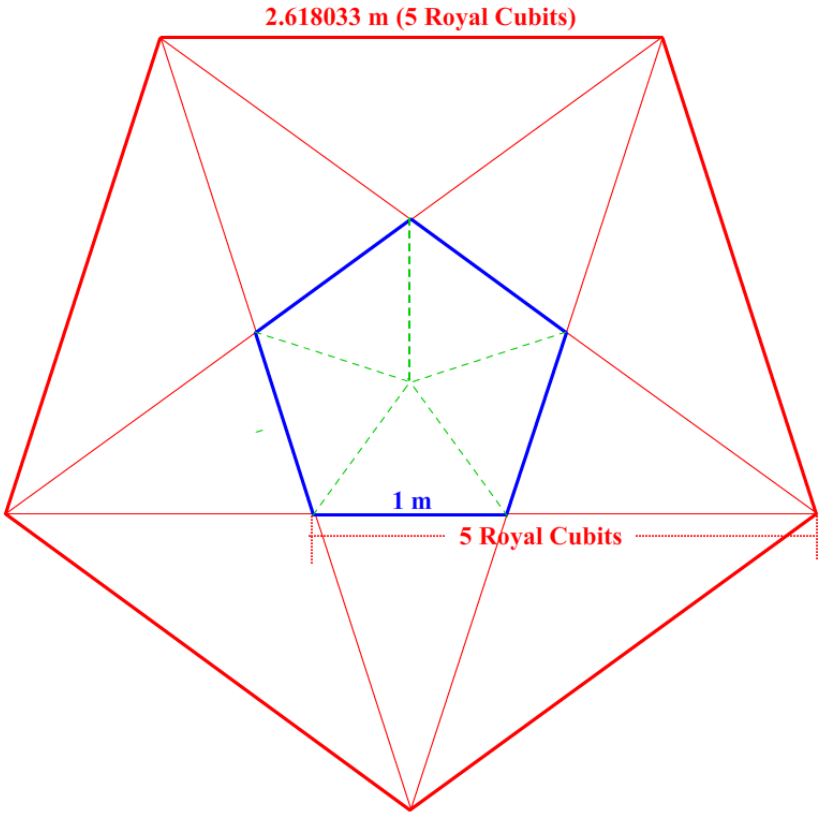


Figure 10