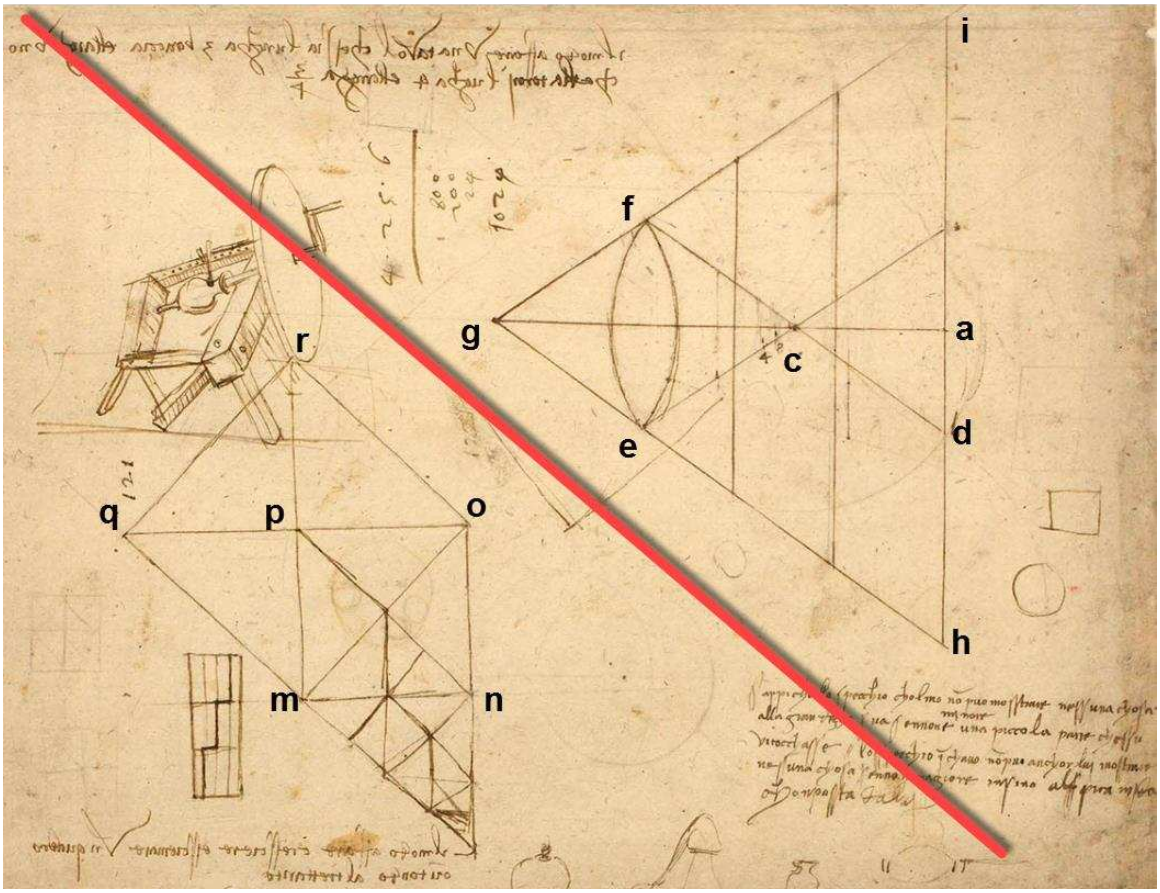


Leonardo and √2

A SINGLE CASE INTERPRETATION

Carl Balowski | January 2024

Abstract

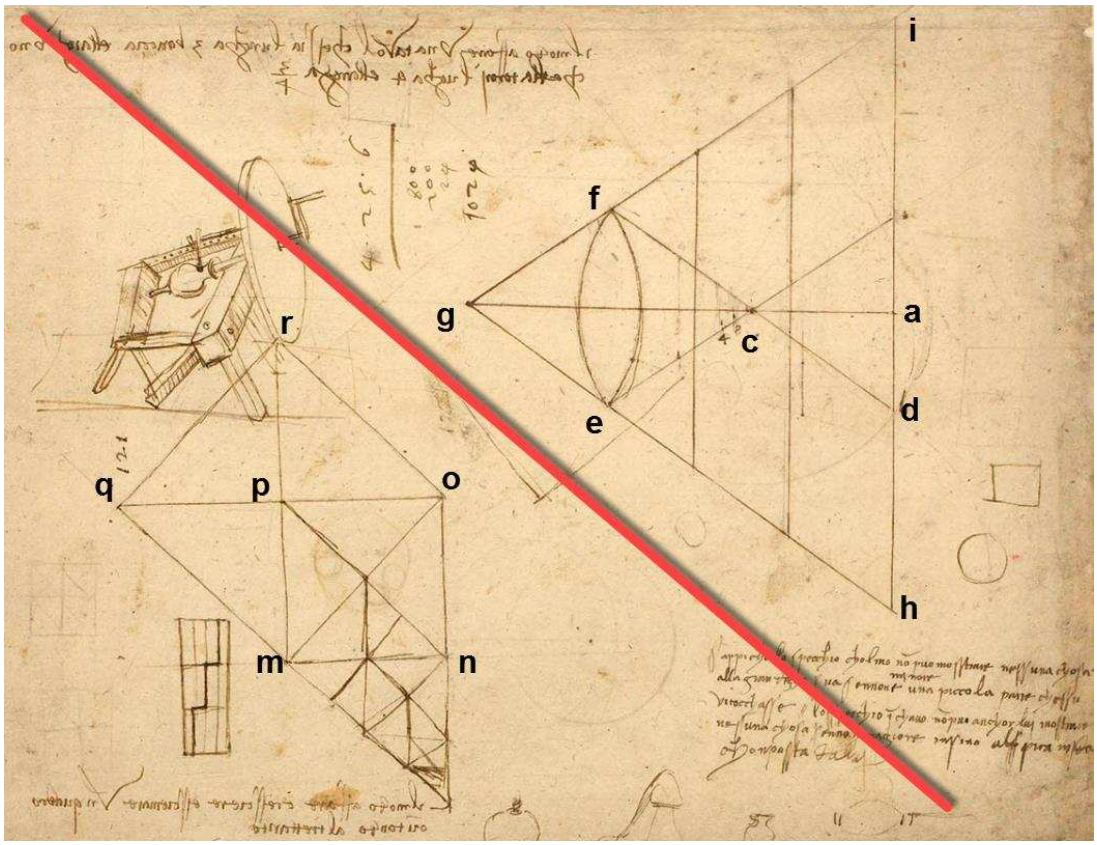
This paper examines a single case of Leonardo da Vinci's profound engagement with Euclidean geometry, focusing on his fascination with the square and its relation to the square root of two. Our analysis centers on the upper figure of page 798 of the Codex Atlanticus (1486), which features two geometric figures, each carefully executed, reflecting Leonardo's intense interest in their properties. These figures exemplify Leonardo's exploration of squares, where he employed the square's diagonal as the side of another, resulting in the replication of the square into diminishing fractals. This method represents a fundamental Euclidean creation, highlighting Leonardo's deep understanding of geometric principles.

Leonardo's application of these principles extended beyond experimental geometry, influencing his artistic, anatomical, architectural, and mechanical design work.[1] His mastery of perspective in paintings like "The Last Supper" in cooperation with Luca Pacioli and his anatomical studies demonstrate his ability to translate geometric concepts into realistic representations and functional designs. Furthermore, his architectural and mechanical inventions showcase his skill in applying precise geometric calculations.

A detailed examination of the upper geometric figure on page 798 of the Codex Atlanticus reveals Leonardo's involved fascination with the square root of two. The lower figure shows how a square's diagonal becomes the side of a larger square, illustrating the concept of $\sqrt{2}$ and $\sqrt{2}^2$. The more complex top figure, upon analysis, also demonstrates a ratio of $1 : \sqrt{2}$, but suggests Leonardo's exploration of the multiplication of $\sqrt{2}$ and $\sqrt{3}$ by a factor of 3. This paper proposes a step-by-step reconstruction of these figures, offering insights into Leonardo's geometric methodology and its potential implications across his diverse body of work.

[1] Duvernoy, S. (2008). Leonardo and Theoretical Mathematics. In: Duvernoy, S. (eds) Nexus Network Journal. Nexus Network Journal, vol 10,1. Birkhäuser Basel. https://doi.org/10.1007/978-3-7643-8728-0_5

Here is page 798 of the Codex Atlanticus dated to 1486. It contains two geometrical figures that Leonardo must have had some keen interest in, for the lines are sharp and each figure has some notation.[2] I inserted the points ***a*** through ***r*** for the purpose of demonstration. As you know, the measure of √2 (1.414…) is the diagonal of any square with the side of 1. With that in mind, notice in the bottom figure that the diagonal $\overline{mo}$ of square $\overline{mnop}$ serves as the side $\overline{mo}$ of the larger square $\overline{morq}$. So that if $\overline{mn}$ = 1, then $\overline{mo}$ = √2 and $\overline{mr}$ = $\sqrt{2}^2$ = 2.



The top figure was a bit more difficult to swing, but after some consideration I intuited that it was related to the √2 in the bottom figure. Indeed I was correct, for the proportion between line $\overline{ah}$ and $\overline{ag}$ is equal to 1 : √2, and from there the construction fell into place. Why Leonardo constructed this figure remains a mystery to me, except for the fact that he is demonstrating the multiplication of the √2 and √3 by a factor of 3. Perhaps his notations say as much. Let's look at one possible reconstruction step by step.

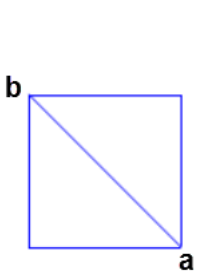
[2] I was unable to obtain a translation of the accompanying notation, but the geometry itself is not too difficult to interpret.

Step 01

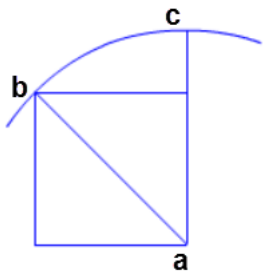
Construct a square and mark its diagonal as line $\overline{ab}$. $\overline{ab} = \sqrt{2}$

Step 02

With your compass on point ***a***, draw an arc to create line $\overline{ac}$. $\overline{ac} = \sqrt{2}$



Step 01



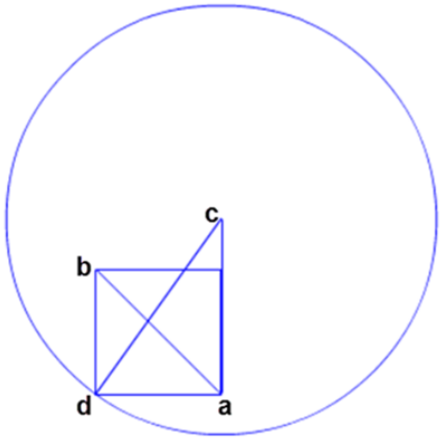
Step 02

Step 03

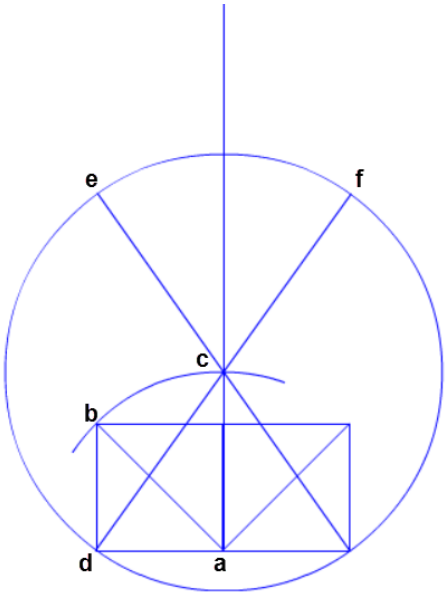
With your compass on point ***c***, use the radius $\overline{cd}$ to draw a circle. $\overline{cd} = \sqrt{3}$

Step 04

Extend line $\overline{cd}$ to intersect with the circle at point ***f***. Do the same on the other side to create point ***e***. Extend line $\overline{ac}$ through the top of the circle.



Step 03



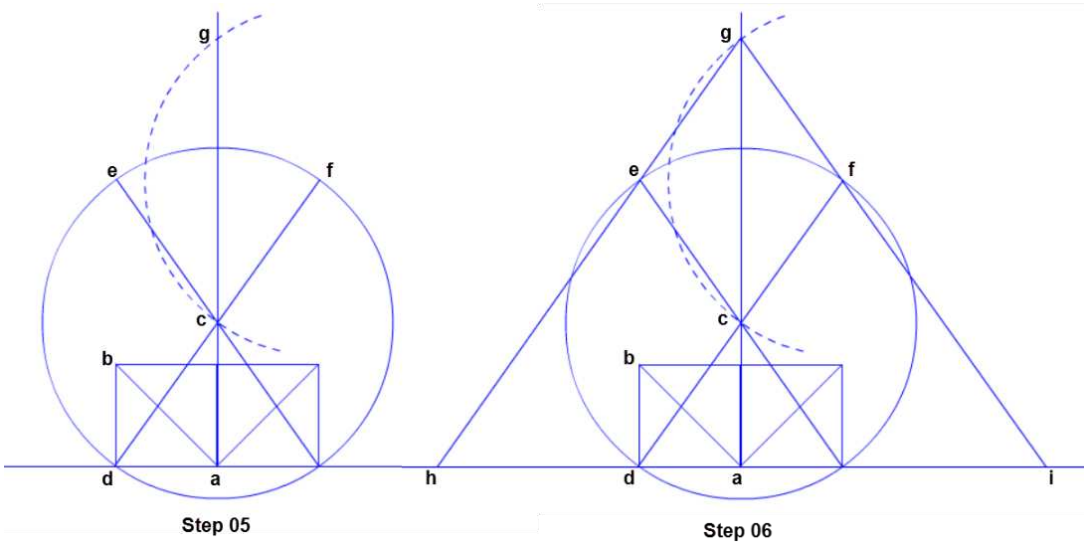
Step 04

Step 05

With your compass on point ***f***, use line $\overline{fc}$ to create an arc that intersects at point ***g***.

Step 06

Create lines $\overline{ge}$ and $\overline{gf}$. Extend line $\overline{ad}$ in both directions. Extend lines $\overline{ge}$ and $\overline{gf}$ to create point ***h*** and ***i***.



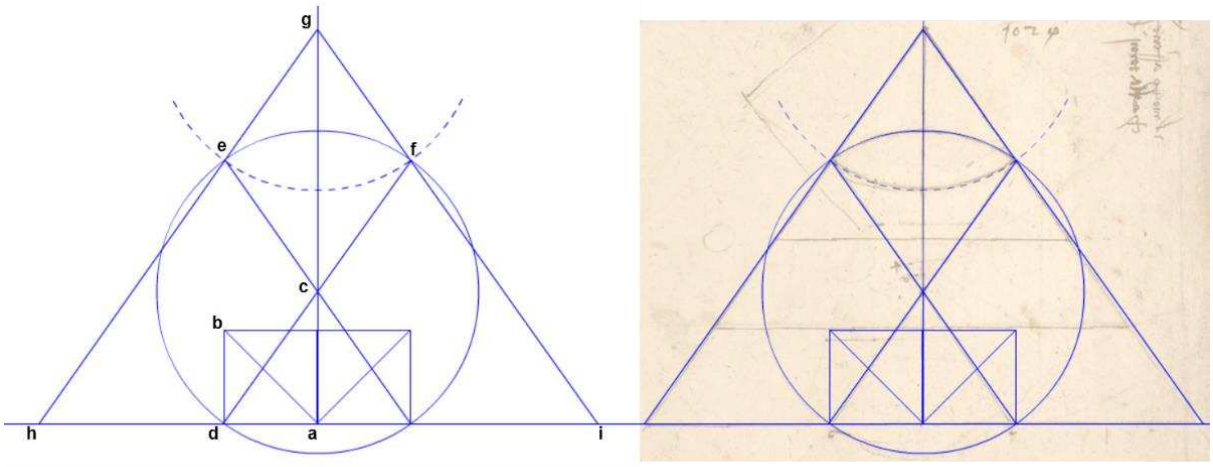
Step 05 Step 06

Step 07

With your compass on point ***g***, use line $\overline{gf}$ to draw an arc through points ***e*** and ***f***.

Step 08

Overlap your construction onto Leonardo's construction. Notice the very high degree of accuracy. Obviously, geometrically speaking, because ours is done on a computer it will be more exact in its representation of the √2 and √3.



Step 07 Step 08

So let's first assign a 'geometric' number value to all of the lines in our construction in order to witness that its foundation is truly based upon the √2 rectangle and its corresponding √3 diagonal (hypotenuse).

$\overline{ad}$ = 1

$\overline{ac}$ = √2

$\overline{dc}$ = √3

$\overline{ef}$ = 2

$\overline{ah}$ = 1 x 3

$\overline{cg}$ = √2 x 2

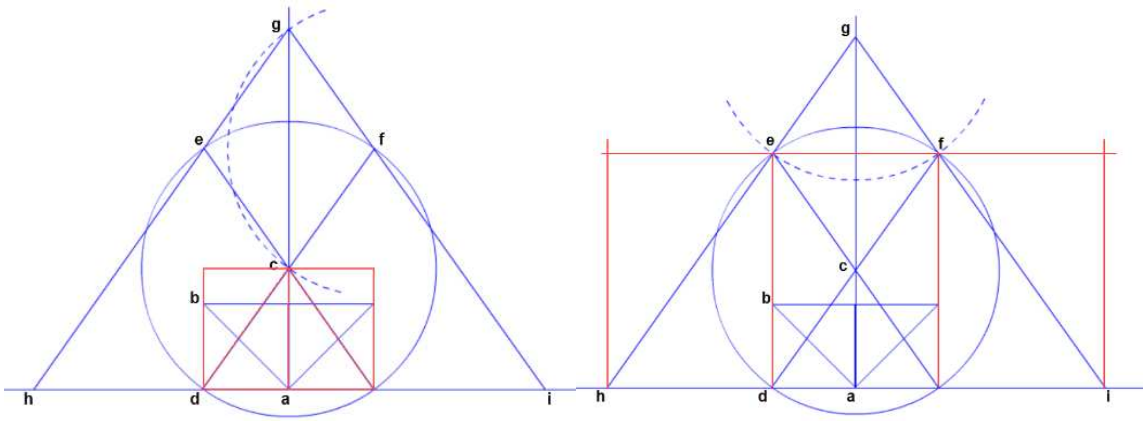
$\overline{ed}$ = √2 x 2

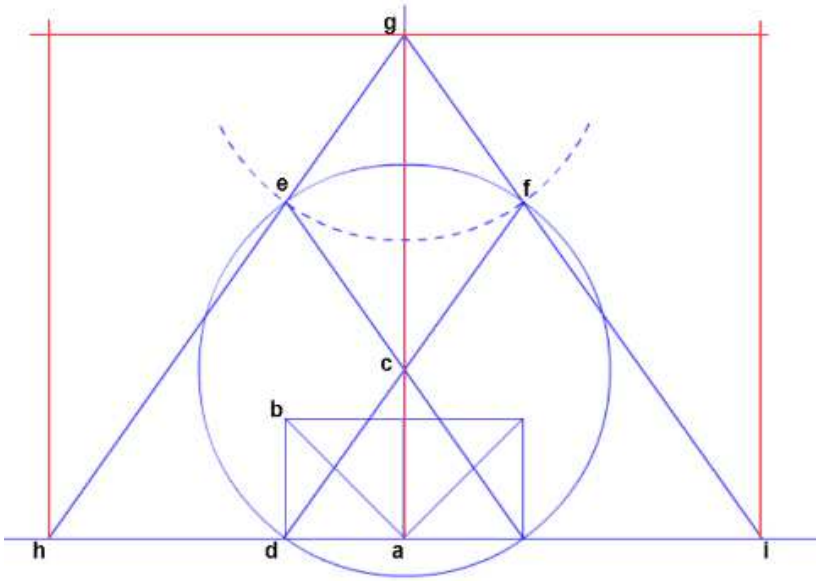
$\overline{ag}$ = √2 x 3

$\overline{df}$ = √3 x 2

$\overline{gh}$ = √3 x 3

Much like his other drawing on the page which is concerned with creating a square which multiplies itself, it doesn't take much to see that Leonardo's focus of this construction was to demonstrate how to increase the √2 rectangle, and its corresponding √3 diagonal, first by a factor of 2, then 3. The obvious crux of the construction was Leonardo's need to use √3 (line $\overline{cd}$) as the radius of a circle which gives birth to line $\overline{ef}$ as a chord.





Leonardo's generation of the √3 is reminiscent of Euclid's very first very proposition — the so-called beginning of his creation — which inherently manifests the √3 through the construction of the equilateral triangle, the most primitive plane surface. Keep in mind that the purpose of Euclid's Elements is to start with a simple axiomatic line (ab) in chapter 1 and proceed methodically (using only straightedge, compass, and logic) to the construction of the dodecahedron in chapter 13. The dodecahedron became associated with Aristotle's *quintessence*, the fifth element — which, because of its mathematical relationship with the pentagon, is itself a manifestation of the Golden Ratio. Pacioli's appellation for the Golden Ratio was the *Divine Proportion*.