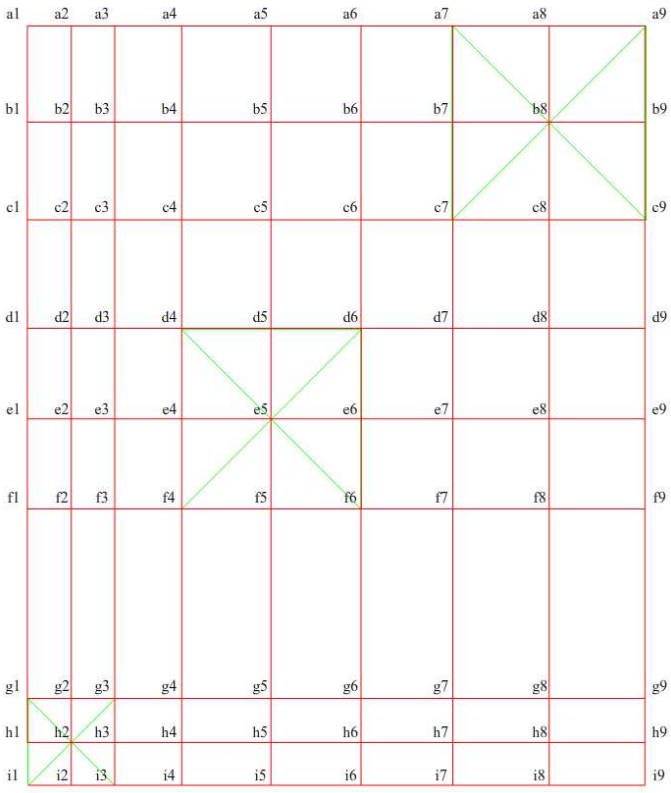


THE GIZA GRID

Geometric solutions to Heliopolitan cosmology.

ABSTRACT

The Giza Plateau, emblematic of ancient Egyptian pyramidal design, serves as an intersection of architectural mastery and Heliopolitan cosmogonical significance. The Great Pyramid's exact orientation towards Heliopolis, the origin of sun god Ra, suggests a deep interplay of architectural intent and spiritual representation based on established Egyptian sighting practices. Investigations by notable Egyptologists imply celestial alignments throughout ancient Egypt, linking them to solstice events and reinforcing their association with Heliopolitan cosmogony. The "Giza Grid", a novel 81-point computational map of the latest survey, offers insights into this complex relationship between theology and geometry, especially the so-called Golden Ratio. A notable intersection on this grid, precisely between the capstone junction of the Khufu and Khafre pyramids, further highlights the importance of 'sighting' during the midwinter sunrise in the pharaonic architectural decisions. Additionally, Hemiunu's mastaba exhibits a deliberate alignment with mathematical constants and celestial events, epitomizing ancient Egypt's fusion of mathematical precision with cosmic reverence. This study seeks to elucidate the intricate nexus of astronomy and geometry in Heliopolitan beliefs, underlining the genius of ancient Egyptian architects in harmonizing monumental constructions with spiritual significance.

Carl Balowski, December 2023

Introduction

The Giza Plateau, the exemplar of ancient Egyptian pyramidal architecture, is not only a testament to the grandeur of Pharaoh Khufu and his successors but also a rich tapestry interweaving mathematical precision with profound Heliopolitan cosmogony involving the creation of the world and the resurrected sun God. The Great Pyramid's (GP) impeccable 45° diagonal orientation towards Heliopolis, the cradle of the sun god Ra, reveals an intricate mesh of architectural strategy and spiritual symbolism, rooted deeply in the age-old Egyptian practices of sighting. The meticulous alignment of the pyramid's SW cornerstone elucidates the pivotal role of the pyramid's apex, a monumental reflection of the √2 and the Golden Ratio, in shaping the entire Giza complex's blueprint. Recent explorations, including those by Dash, Lehner, Belmonte and Magli, have hinted at celestial alignments involving the pyramids of Khufu and Khafre. These alignments, connected to solstice events, further entwine the pyramidal structures with Heliopolitan beliefs, emphasizing the pyramids' roles as conduits between the terrestrial and the cosmic. The sacred Benben stone, central to this cosmogony, emerges as an emblem of creation, symbolizing the unison of divinity, nature, and humanity.

The intriguing "Giza Grid" (Figure 1) — a new computational 81-point grid-map based upon the most recent survey provided by Glen Dash (Figure 2)— offers a modern lens to understand this ancient confluence of theology and geometry.[1] By juxtaposing the grid's mathematical rigidity with Egypt's mythological fluidity, we delve deeper into the very essence of Egyptian cosmogony. A particularly significant point on the grid at the center junction of Khufu and Khafre's pyramids, provides insights into the spiritual significance of 'sighting' the midwinter sunrise over the cornerstone in the pharaohs' design choices. Meaning that an observer aligning themselves exactly between the cardinal junction of the pyramid's capstones at B5 could view the Winter Solstice sun rise precisely over the cornerstone C7 of Khufu's pyramid, indicating that the extended center and corners of each pyramid acted as a sort of graticule for positional observation (Figure 3). Furthermore, a keen examination of Hemiunu's mastaba, a fine specimen of Fourth Dynasty architectural prowess, also showcases a masterful fusion of mathematics, cartography, and religious beliefs. The structure's alignment with the square root of 5 (√5) — the basis of the golden ratio — and its purposeful orientation to capture the winter solstice sunrise over the cornerstone of Khufu's pyramid, is a testament to ancient Egypt's marriage of mathematical ingenuity with a deep-seated reverence for the cosmos. The intersection of celestial bodies with architectural structures within the realm of ancient Egyptian studies has remained a captivating and intriguing subject of exploration. The field of Egyptology, with its multidisciplinary approach, has meticulously uncovered the intricate interplay between cosmology, religious beliefs, and monumental architecture during the era of ancient Egypt.

1 Dash, "Where, Precisely, are the Three Pyramids of Giza?" (2017). Revision of May 5, 2017 used data from the Glen Dash Foundation survey of 2015 (GDFS 2015) to revise the formula for converting Petrie's station points to Giza Plateau Mapping Project (GPMP) coordinate points.

This paper embarks on a journey to illuminate a profound confluence of geometry and astronomy within the Heliopolitan cosmogony at Giza, for the Giza Plateau stands as an eloquent testimony to the brilliance of ancient Egyptian architects and their profound connection to the spiritual realm. An illustrative methodology is provided at the end of this paper.

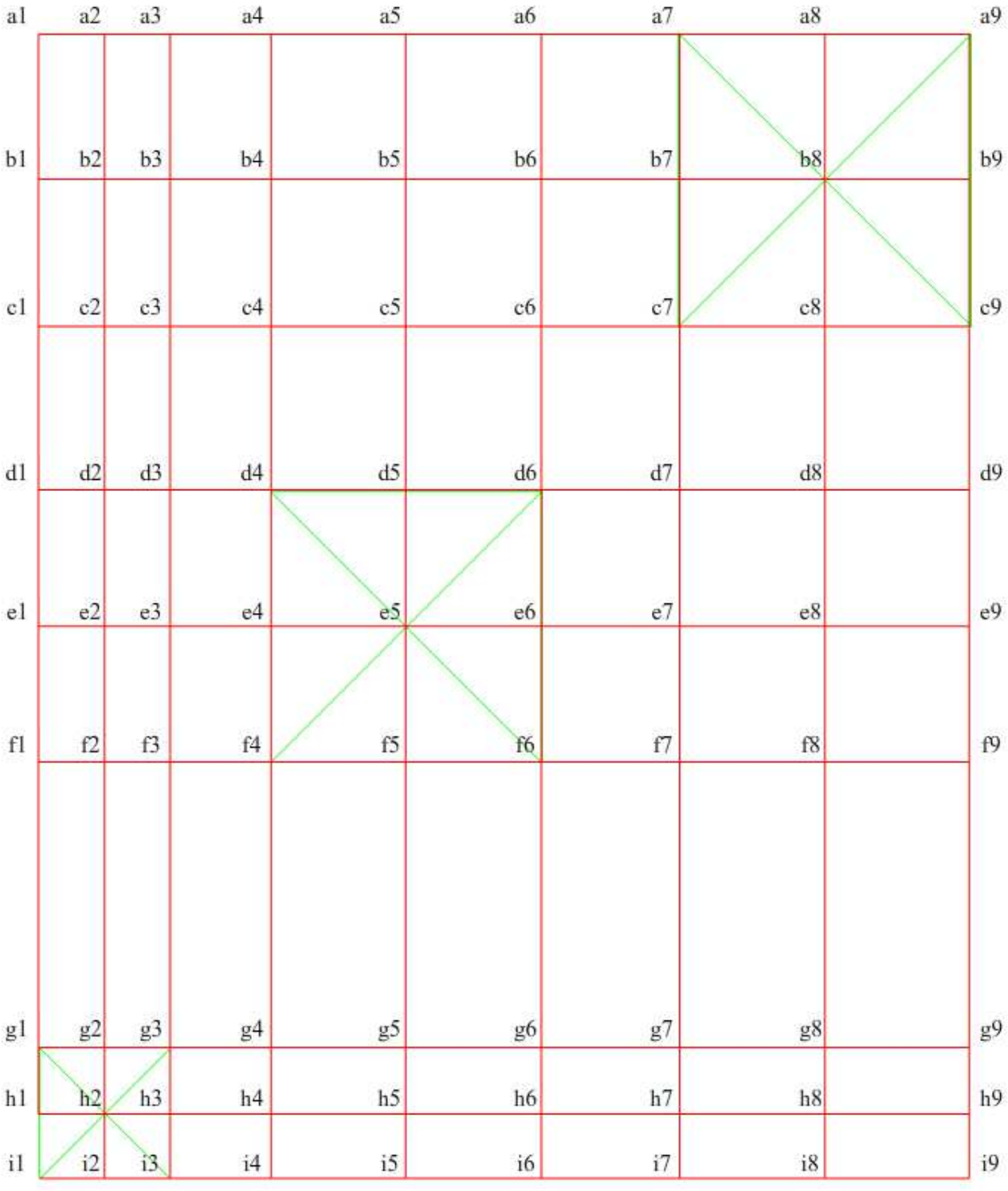


Figure 1 - The Giza Grid.

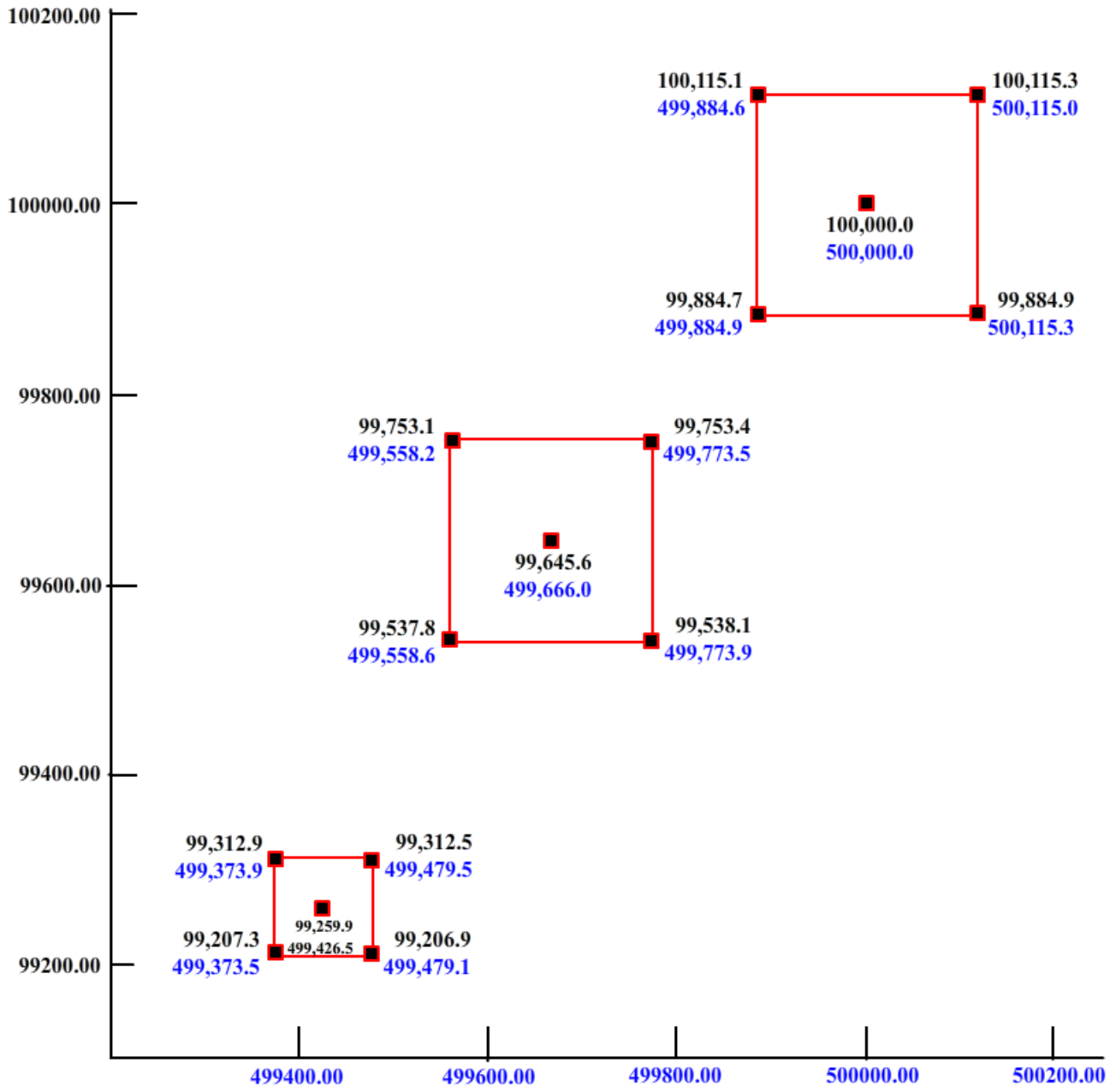


Figure 2 – GDFS 2015 metered coordinates (based on Dash's 2017 revision).

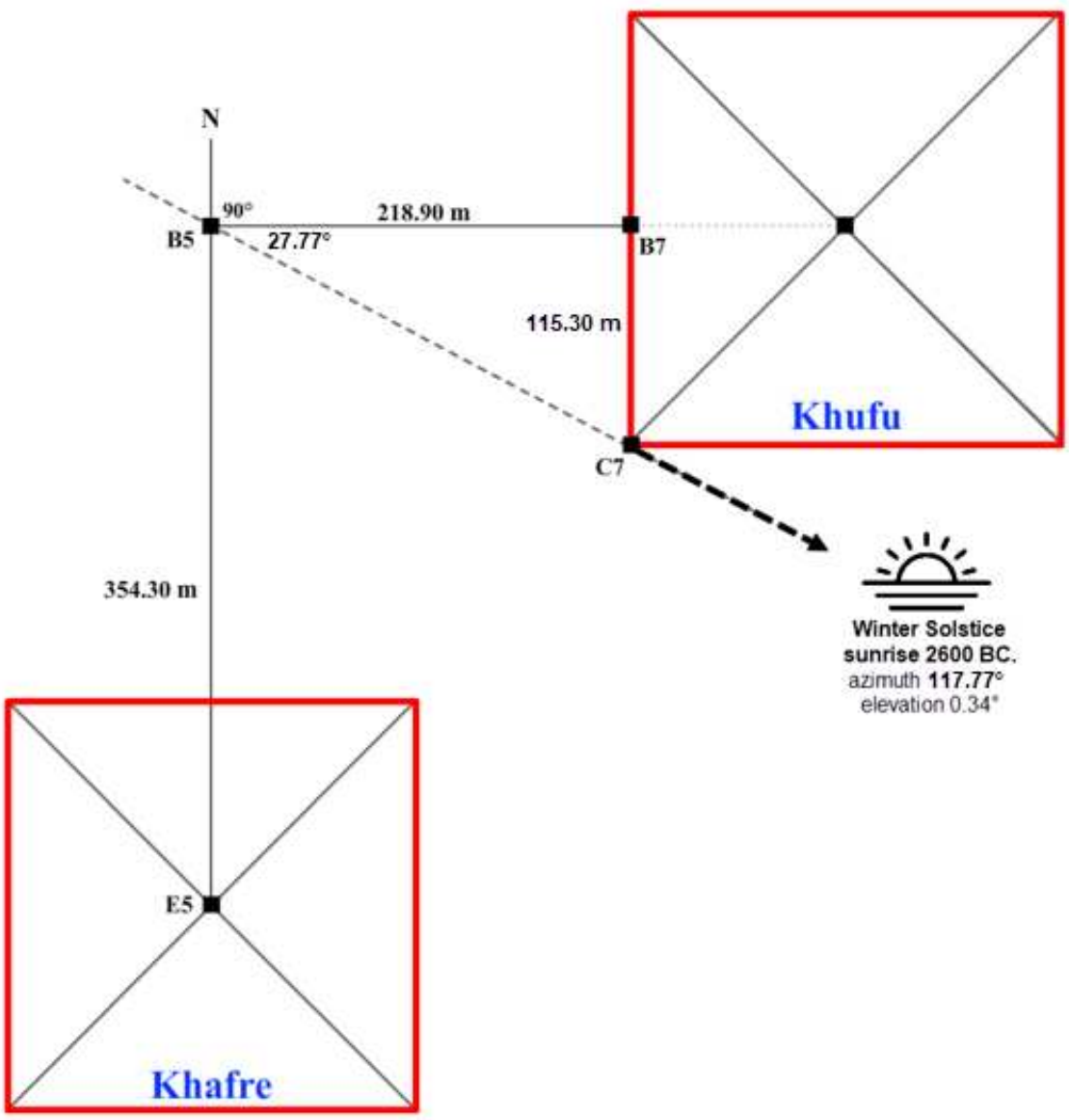


Figure 3 – The pyramids as sighting guides for the Winter Solstice sunrise, ca 2600 BC.

New paradigm

The year 2011 marked a significant juncture in the exploration of Egyptian archeoastronomy, as scholars Glen Dash and Mark Lehner published a captivating article under the auspices of the American Research Center in Egypt (AERA). Drawing insights from the Giza Plateau Mapping Project (GPMP), Dash and Lehner presented a compelling hypothesis, postulating that the architects of the Giza pyramids might have ingeniously employed an observation point located on the colonnade of the Sphinx temple. This strategic viewpoint, according to their proposition, allowed these ancient architects to witness the mesmerizing spectacle of the Summer Solstice sunset circa 2500 BC. This celestial alignment, positioned between the grand pyramids of Khufu and Khafre, undoubtedly stimulates curiosity. However, the persuasive potency of this argument is marred by the lack of robust astronomical evidence to substantiate their claims. It is worth noting that Dash and Lehner drew inspiration from the research of J. A. Belmonte and Giulio Magli, who had thoroughly delved into the intricacies of the Egyptian Calendar and astronomy. Following the long-accepted hypothesis that Karnak

is aligned to the Winter Solstice sunrise ca 2000 BC, their research hinted at earlier Winter solstice observations conducted at specific temple structures, including Giza.[2] An exemplary instance includes the dramatic sunset over the northwest corner of Snefru's Bent Pyramid, viewed from the central vantage point of its accompanying temple (Figure 4).[3]



Figure 4 – Winter Solstice sunset over the cornerstone of the Bent Pyramid (ca 2011).

From a bird's eye view all pyramids seem a geometric marvel, presenting themselves as a square intersected by two prominent diagonals. Yet, Khufu's Great Pyramid (GP) of Giza, the largest of the three pyramids on the plateau, encompasses a vast range of other fascinating features and intricate details, each telling a story of its own. One of the most fascinating yet mostly overlooked aspects of the GP is its impeccable 45° diagonal orientation (Figure 5) which pinpoints the ancient city of Heliopolis, the renowned cultic epicenter believed to be the 'site of creation', dedicated to both Atum and Ra the sun god. This is not merely a result of chance or a cursory design decision but stands as a testament to the Heliopolitan planning and profound spiritual significance interwoven into the fabric of the pyramid's architecture.

[2] See AERAgram, VOL. 12 NO. 2 and Magli, G. (2013). Architecture, Astronomy and Sacred Landscape in Ancient Egypt. United States: Cambridge University Press, p. 179-185.

[3] Sunset on the winter solstice observed from the center line of the lower temple of the Bent Pyramid at Dahshur. Due to the Earth's changing tilt, the sun would have set a little more than one sun disk's diameter to the left in 2500 BC, clipping the pyramid's northwest corner. Photo by J. Belmonte.

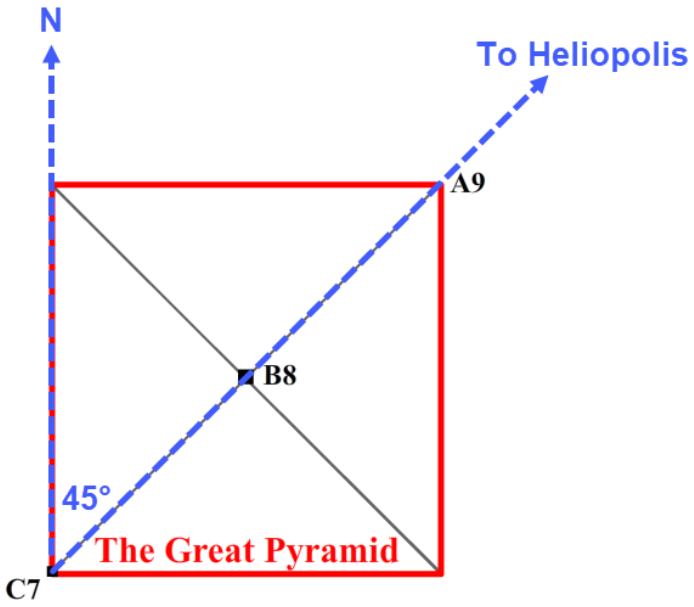


Figure 5 – The Great Pyramid's exact √2 orientation towards Heliopolis

Now, allow your mind to traverse back in time and assume the role of Hemiunu, the eminent architect and priest tasked with supervising the inception and construction of the iconic pyramids on the Giza Plateau. In this august position, the responsibility of strategizing the GP's capstone (point B8) is paramount. This pivotal point not only dictates the foundational blueprint for the placement of every other pyramid on the plateau, but is also symbolic of the pyramid's overarching design, particularly its renowned slope in accord with the Golden Ratio and its primary burial chamber in accordance with the √5 (Figure 6).[4]

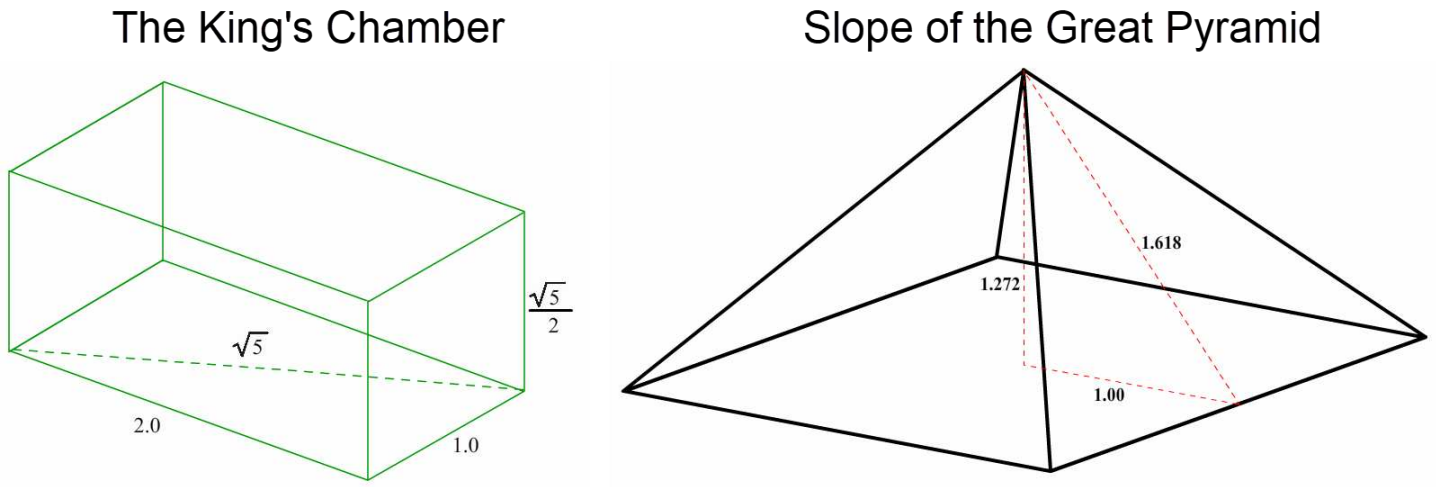


Figure 6 – The Golden Ratio and √5 in the Great Pyramid's King's Chamber and construction slope.

[4] The Great Pyramid's construction using the Golden Ratio and √5 is well documented online and will not be discussed here.

Highlighting the significance of Khufu's capstone at B8, even contemporary studies, such as the detailed mapping project of the Giza Plateau undertaken by Glen Dash, relied principally on the GP's apex as the survey's primary control point. Through the ancient and esteemed practice of 'sighting', a surveyor would position themselves so that Heliopolis, distant in the river valley, was perfectly framed by a surveyor's rod located on the control point B8. This exacting procedure allowed the surveyor to demarcate the location of Khufu's southwest cornerstone at C7, no matter how distant from B8, ensuring an amazing alignment of the pyramid's diagonal with Heliopolis at a precise 45° angle relative to True North.[5]

Once cornerstone C7 was established, the northeast cornerstone at A9 could be determined, ensuring it was both equidistant and in perfect alignment with the southwest cornerstone and the pivotal control point B8. This created a resolute and spiritually significant diagonal directly pointing towards the revered city of Heliopolis. Subsequently, marking the northwest and southeast cornerstones was essentially a geometric formality of ensuring equidistant measurements to achieve a perfect square. From this intricate process, it will become clear that the positioning of the control point at B8 and the southwest cornerstone at C7 were done in tandem to not only determine the size, orientation, and placement of the GP, but indeed the very foundational blueprint of the entire Giza complex. Thus, it can be reasonably posited that Khufu's southwest cornerstone at C7, akin to his capstone, held an exalted, perhaps even sacrosanct, significance in connection to the cult of Ra at Heliopolis.

The Benben Stone

At the crux of comprehending the profound import of a pyramid's capstone lies the Heliopolitan 'Benben stone', the intricate theological bedrock of Heliopolitan cosmogony. Rooted in the veneration of Atum and Ra (the creator / solar deities), this cosmogony forms a pivotal *basis* in our understanding of the ancient Egyptian worldview. Atum, an entity often depicted adorned with the regal dual crown emblematic of Upper and Lower Egypt, emerged from the primordial waters referred to as "Nu." The etymology of "Atum" (which interestingly seems cognate with Adam) itself bears significance, carrying connotations of completeness, negation and becoming —an intriguing paradox that accentuates his role as the embodiment of self-evolution.

Through his creative act, Atum initiated a cascade of creation, ushering forth the birth of several deities. This foundational event triggered an intricate chain reaction, signifying the genesis of light and the cosmos. This portrayal of Atum's self-creation underscores his representation as a symbol of both modification and universal completeness—a deity persistently existing in a state of perpetual becoming. It is pertinent to note that while Atum does not exclusively

[5] Ancient Heliopolis is 14.75 miles away from the Great Pyramid. According to Google Earth Pro a heading of exactly 45.00° from C7 of the Great Pyramid lands a mere 28 feet west of the last remaining obelisk at ancient Heliopolis. On the accuracy of using Google Earth Pro (GEP) heading information for archaeoastronomy see William F. Romain 'Notes on the Accuracy of Google Earth Pro Heading Information (2022)'. On the discovery and feasibility of the 45° azimuth between Giza and Heliopolis see Magli 'Archaeoastronomy: Introduction to the Science of Stars and Stones.' p. 169-171. As a note, Magli's survey data of the 45° azimuth did not employ GEP or Dash's survey of the Plateau. My finding is that the 45° azimuth between the GP and Heliopolis is even more precise than what Magli proposes.

assume the mantle of a solar deity, his symbiotic relationship with the sun god Ra amplifies his stature as a representation of the life-affirming essence embodied by the sun. This fusion magnifies the intricate interconnectedness and fluidity characterizing Egyptian deities, allowing them to embrace multifaceted roles. The Pyramid Texts, an intricate corpus of ancient Egyptian funerary texts, establish a profound resonance between Atum, humans and the act of creation, where the Benben stone emerges as an emblematic point arising from the primeval waters of "Nu." This sacred stone occupies a position of profound significance within Atum's divine act of invention, symbolizing the very locus from which the fabric of existence emanates. The imagery of Atum's symbolic grip upon his phallus conveys dual motifs of creation and self-awareness, encapsulating the cosmic narrative within a profound gesticulation.

The profound significance of the Benben stone is further enriched by delving into its intricate association with the avian deity Bennu (Figure 7) —a deity that intriguingly foreshadows the legendary mythos of the phoenix. Bennu, existing as an alternate symbol of Atum, carries a multifaceted connection with the Benben stone and the celestial sun. This connection is accentuated by the shared phonetic resonance of the term "weben," encapsulating the very essence of the sun's glorious emergence from the boundless watery horizon. Notably intertwined with the annual inundation of the Nile, Bennu's narrative extends to him soaring above the primordial waters, mirroring the cyclical rejuvenation of the land. It is akin to the initial rays of the sun that delicately grace the apex of the emerging Benben stone, embodying the emergence (or rebirth) of light and life. This parallel narrative culminates as Bennu alights upon the very first spot of dry land to break through the ancient waters—an emblematic reflection of creation and new beginnings.

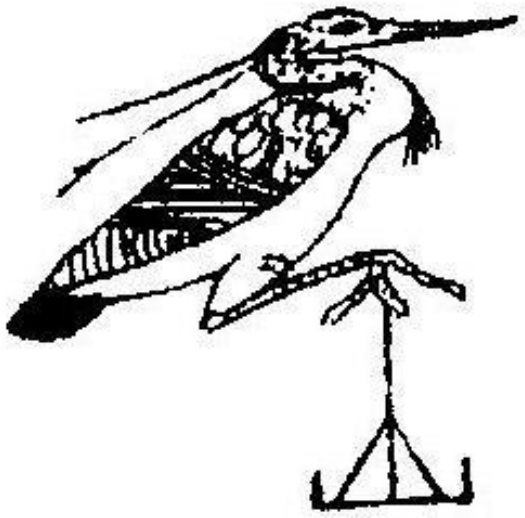


Figure 7 – Bennu upon the first reed emanating from the Benben stone.

As a poignant embodiment of cosmic symbolism, the intricate fusion of the Benben stone, Bennu, and the sun attests to the nuanced interplay between mythology, nature, and divinity and the confluence of the Egyptian deities Atum and Ra, finds its intricate manifestation at least by the 5th dynasty. This convergence reverberates most prominently within the sacred precincts of Heliopolis, the venerated city that stood as a bastion of celestial contemplation and theological exploration of life and death.

Heliopolis, the center of creation

Heliopolis (in Greek), known as "Iunu" in the ancient Egyptian texts and ‘On’ in Hebrew, held an esteemed position as a center of theological discourse and astronomical observations. This city became the crucible where the divine narratives of Atum and Ra melded to form a captivating tapestry of creation and cosmic order. Atum, the primeval deity symbolizing self-creation, and Ra, the radiant sun god heralding light and life, coalesced in this celestial hub, casting an everlasting shadow of their divine synergy. Later, the concept of Zep Tepi, the "First Time," encapsulated the pivotal moment of creation—a belief that the initial rays of the sun gently touched the Benben stone, signifying the dawn of existence. The interplay between Atum's primordial emergence and Ra's eternal radiance laid the foundation for the Egyptian understanding of the cosmos and the cyclical patterns of life, death, and rebirth. This profound interweaving of divine narratives was at the core of the ancient Egyptians' perception of reality.

Within the sacred precincts of Heliopolis, the Temple of Ra emerged as a tribute to the synthesis of Atum and Ra. This monumental complex embodied the cosmic partnership, with the Benben stone at its core, representing Atum's creative nucleus, and the worship of Ra encompassing the celestial order. The sun god Ra, synonymous with vitality and sovereignty, personified the cycle of the sun, its daily ascent and descent as captain of a solar boat (Figure 8).



Figure 8 – The cyclic rising and setting of Ra and Bennu in his solar boat.

Truly the Old Kingdom -- characterized by monumental construction and regal grandeur -- witnessed the embodiment of these deities in the earthly rulers. Pharaohs of this era often associated themselves with Atum or Ra, establishing their divine right to rule by aligning with the sun god's majesty. The pyramids, newly formed architectural marvels of the Old Kingdom, were conceived as a conduit to facilitate the pharaoh's ascent to the celestial realm, echoing the divine ascent of Ra and the creative emergence of Atum as the Benben of Heliopolis. The apex of these pyramids, crowned by the

pyramidion, embodies a symbolic conduit that unites the monumental edifice with the sacred essence encapsulated within the Benben stone. These pyramidions, some adorned with inscriptions of religious symbols and royal titles, transcend mere visual ornamentation. If gilded and polished, they were believed to possess the ability to reflect the sun's resplendent rays —an awe-inspiring manifestation of divine luminescence. The enduring legacy of the Benben finds palpable expression in the pyramidion of Amenemhat III—a monument crafted from enduring black granite that silently attests to the eternal interplay of the sacred and the architectural (Figure 9).



Figure 9 – A pyramidion in the form of the Benben stone.

The Enigma of the Giza Plateau

Revisiting the hypothesis, the alignment of the pyramids on the Giza Plateau with celestial events resounds with profound echoes within the intricate tapestry of Heliopolitan beliefs. The pyramidions perched atop the pyramids of Khufu and Khafre transcend mere markers, metamorphosing into geometrically 45° sacred conduits that bridge the earthly structures with the cosmic significance encapsulated within the Benben stone at Heliopolis. Khufu was intricately linked with Ra, and inaugurated an era often dubbed the "sons of Ra," with his pyramid functioning as the embodiment of the *Akhet Khufu* or the "horizon of Khufu." From here we can see that the 45° alignment of his pyramidion not only established an unbreakable sacred bond with Heliopolis but also enveloped the concepts of mortality and creation within the cyclical rhythm of solar rebirth.

Pyramids are ‘a’ square

Venturing deeper into the heart of the enigma, we encounter the abiding allure of the geometrical construction underpinning the pyramid—a motif that resonates with both architectural form and mythological substance. This geometric emblem engenders a profound revelation: regardless of the slope angle defining it, all pyramids invariably manifest -- when contemplated from a centered aerial perspective – a flat square adorned with its two 45° diagonals, birthing five distinctive points, namely its four cornerstones and its single center point (capstone) on a plane (Figure 10).

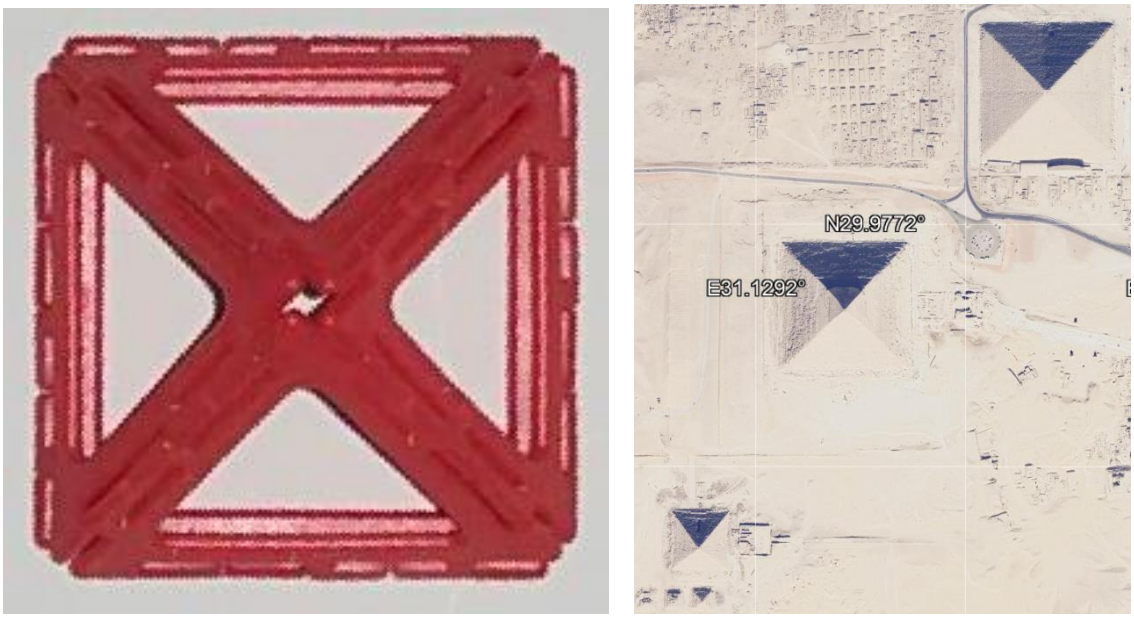


Figure 10 – As seen from above, a pyramid takes the form of a square with two diagonals.

Within this square capstone, or Benben, is a center point formed by the pyramid's diagonal. This point is the essence of Heliopolitan cosmogony, and it takes us on a conceptually creative journey as we envision grasping it and elevating it perpendicularly, as if it was emerging from its two-dimensional plane, like the Benben from its primordial abyss. The pharaonic act of 'raising' a pyramid mirrors the creative act of Atum and the metamorphoses of a flat square into a tangible, three-dimensional pyramid (Figure 11). As a manifestation of the so-called 'Word of God', it's a progression from design to construction, from ideal to reality.



Figure 11 – Pulling a square up from the ground to create a pyramid.

The extent to which the center-point is elevated determines the slope angle of the resultant pyramid. The higher we elevate the center, the steeper the slope. However, what is most intriguing is that no matter how high we elevate the capstone, the centered aerial perspective never changes, for it will always appear as a flat square with two 45° diagonals.

The Great Pyramid was elevated to precisely create a slope in accordance with the golden ratio! As intimated, this geometric metamorphosis finds a compelling parallel within Atum's Benben, a symbolic, emerging point that heralds the dawn of existence from the enigmatic depths of the primordial abyss. The ascension of the Benben was the first act of creation and it is intriguing that the builder of the Great Pyramid decided to memorialize this act in accordance with the golden ratio. This mythic emergence resonates with the geometric transformation, wherein the flat square ascends – as if elevated by Atum – into the sunlit pyramidal realm of three-dimensionality, an allegorical underpinning that reverberates in both the geometric representation and the mythological tapestry connecting the realms of the divine, the cosmos, and human perception.

However, for the ancient Egyptians, the Benben stone's role transcended mere allegorical interpretations. It embodied the geometric core of creation, resonating through the mathematical transformation of a basic square into the regal geometrical edifice of a pyramid. This metamorphosis drew symbolic parallels to the legendary ascent of the Benben from below the primordial featureless waters into the voluminous realm of Ra. Standing at this crossroad of cosmogony and mathematical intrigue, we endeavor to decode the profound significance ancient Egyptians found at the juncture of architectural marvels and deep-seated metaphysical tales. Therefore, our quest is further intensified by evidence from Pharaoh Khufu. Archaeological discoveries, particularly his solar boat, lend credence to the belief that Khufu's narrative is inextricably linked to his anticipated rebirth from the Duat—a dimension symbolic of the afterlife—and his radiant reincarnation as Atum-Ra, the venerated solar deity. Such chronicles not only underscore his profound understanding of the symbiotic relationship between earthly mortality and cosmic immortality, but also illuminate his intrinsic desire for an unbroken journey beyond the terrestrial realm.

Perennial solar rebirth

A pivotal tenet in this cosmic narrative revolves around the concept of the sun's cyclical demise in the western horizon, only to emerge anew in the eastern firmament. Beyond the daily cycle, this solar rejuvenation takes on a grander significance during the solstices, epitomizing the sun's annual cycle of death and rebirth. The summer solstice marks the zenith of Earth's axial tilt towards the sun, resulting in an extended duration of daylight—symbolic of life's triumph over darkness. Conversely, the winter solstice represents Earth's greatest tilt away from the sun, culminating in an extended period of nocturnal obscurity—an allegory of life's temporality in the face of adversity. This cyclical pattern then spirals forward to renew itself over a span of 365.24 days, an enduring dance echoing the cosmic rhythm of existence. Within this celestial choreography lies a profound query: what does the first sunrise on the *day following the winter solstice* symbolize if not the embodiment of Zep Tepi—the primal creation, the annual return and "birthday" of Atum's Benben—the pinnacle of all existences, which emerged together with the sun from the watery, primordial depths of an underworld?

In essence, this post-solstice sunrise stands as a resurrection of resurrections, or a metaphoric "birthday of birthdays." The very act of observing this sunrise is synonymous with a tribute to Atum-Ra, the celestial patriarch—his reawakening orchestrated on the heels of the darkest juncture in the yearly cycle. Embedded within this celestial narrative is the profound symbiosis between Khufu's expected resurrection and Ra's annual resplendent emergence. The solar boat's discovery serves as a testament to the ancient Egyptians' resolute belief in the harmonious continuity of life beyond the mortal realm. It crystallizes the ancient civilization's unwavering yearning for an afterlife that mirrors the cyclical revival of the sun—a timeless allegory that intertwines the cosmos with earthly existence. As we stand at the threshold of the winter solstice's dawn, we are beckoned to partake in a commemoration with the ancient Egyptians that echo the timeless sentiment: the sun may set, but it shall always rise anew—a testament to the enduring human spirit that resonates with the perpetual cycles of nature and the act of 'becoming'.

What is the Giza Grid?

Few landscapes have captivated the historian's imagination as profoundly as the Giza Plateau, home to the Great Pyramids of Egypt. One specific focus that merits consideration is the notion of a "Giza Grid," a novel computational overlay that emerges from recent meticulous surveying data. This paper's concept of the Giza Grid (Figure 1) has been computed using Glen Dash's high-resolution survey data published in 2016 (Figure 2).[6] Simple in concept, the Giza Grid is an 81-point grid-map generated by extending graticule lines from all five points of each of the three pyramids.

In the lexicon of cartography, vertical lines thus generated are termed 'Eastings,' while horizontal lines are identified as 'Northings.' The intersections of these Eastings and Northings coalesce into what are termed 'Grid Points,' 81 in total. The specific coordinates of each Grid Point are ascertained by referencing either its Northing or Easting origin point. To quantify the spatial relationships further, the calculated distance between any two Grid Points, either on a horizontal or vertical axis, is termed a 'Grid Line.' In the case of a vertical Grid Line, the calculated distance is obtained by subtracting the coordinates of its most southerly Grid Point from those of its most northerly counterpart. Conversely, in the context of a horizontal Grid Line, the calculated distance emerges from the subtraction of its most westerly Grid Point from its most easterly point. Three crucial observations emerge from this methodology. First, the five points of each pyramid manifest an instrumental significance within the geometric layout of the Giza Plateau. Second, the grid generates several rectangular ratios, imbuing the spatial relationships among the three pyramids with properties that can be described as 'Euclidean' in nature. Last, the NE and SW cornerstones of Khufu's and Menkaure's pyramids (A9 & I1 respectively) serve as defining markers for the overall dimensions of the grid.

The pressing inquiry that ensues relates to the historically attested use of specific mathematical principles, such as the Golden Ratio, √2 and √5, in ancient Egyptian architecture and metrology, notably in the mastaba and pyramid designs of

[6] Dash, "Where, Precisely, are the Three Pyramids of Giza?" (2017)

Hemiunu and Khufu. The question is whether we should expect these mathematical tenets to extend to the configuration of the Giza Grid itself. Given the consistent application of these ratios and the √2 : 1 metric in their architectural enterprises, it seems almost inevitable that these principles should manifest within the Giza Grid. Indeed, if they were absent, it would pose an intriguing anomaly, inviting further exploration into the extent and limitations of mathematical knowledge and its application in ancient Egyptian civilization. Thus, the Giza Grid presents itself as a multifaceted instrument of analysis, with implications that extend beyond mere cartographic utility. It holds the potential to function as a lens through which we may gain epistemic insights into the intricate tapestry of ancient Egyptian civilization, where geography, mathematics, and cosmogony were woven together in a harmonious tableau. It is this interplay, mirrored in the grandiose designs of antiquity, that the Giza Grid invites us to explore more deeply. The critical question that arises, however, is to what extent we can place our reliance on Giza Grid in deciphering the underlying intentions of the ancient builders.

While we are equipped with increasingly sophisticated tools and methodologies that enable us to probe the past with greater precision, we must remain cognizant of the inherent limitations that accompany any analytical framework. The ancient Egyptians' world view, steeped in a complex cosmological narrative as seen above, undoubtedly influenced their architectural and geographical choices. As we seek to unravel the mysteries of this ancient civilization, the Giza Grid represents a valuable, perhaps definitive, step in our ongoing quest for understanding. It is incumbent upon us to wield this tool judiciously, while maintaining a critical perspective that is attuned to the nuances and complexities of this fascinating chapter in human history. Indeed, one of the most fascinating intersections of ancient culture and scientific ingenuity might be found in the juxtaposition of the Egyptian creation myths and their profound interest in astronomy and geometry. While these two facets of Egyptian civilization may seem disparate at first glance, a closer investigation reveals a confluence of ideals that may be better understood through empirical methods like this paper’s Giza Grid. Specifically, one might ask: is it possible to analyze and reconcile these Egyptian beliefs and mathematical accomplishments by employing measured ratios found on the Giza plateau? The answer is a nuanced yes, though with certain caveats—primarily the introduction of what is known as "Percentage Error" into our calculations.

The Concept of Percentage Error in Empirical Analysis

Before delving into the intricacies, it is pertinent to understand the role of percentage error in this investigation. Simply put, percentage error is a numerical expression of the discrepancy between a measured value and an established theoretical or true value. For example, if one is dealing with the ratio of the diagonal to the side of a square, the ideal mathematical value would be √2:1. When comparing measurements derived from, say, the Giza Grid, to this ideal value, any difference is quantified as percentage error. Expressed mathematically *percentage error* = $\frac{E-T}{T}\ x\ 100$, where E = the experimental value and T = the theoretical (ideal) value.

When we consider the Giza plateau, particularly through the lens of Glen Dash's meticulous survey, it becomes evident that the experimental values we measure are not 'perfect' replicas of the ideal mathematical values. These deviations can arise from a plethora of factors, including but not limited to the limitations of ancient or contemporary surveying tools, nuances in techniques, calculation rounding, construction imperfections, and other miscellaneous inaccuracies. Consequently, assessing percentage error allows us to attribute a quantitative dimension to the divergence between what has been empirically observed and the platonic ideal posited by mathematical theory. Percentage error serves as an invaluable barometer for the accuracy of our findings. For example, some think that a small percentage error—within the realm of 1%—suggests that our measurement somewhat closely aligns with the mathematical ideal. On the other hand, a larger error, in the vicinity of 10% or even 5%, is an unequivocal indicator that the measurements substantially deviate from the ideal. This could potentially signal inaccuracies in methodology or even flaws in the underlying hypothesis.

Interestingly, the architectural marvels on the Giza plateau offer a remarkable precedent for precision. The North-South orientation of both the Great Pyramid and Khafre's pyramid exhibits a stunningly low percentage error of less 0.1%. Glen Dash further illustrated that the mean value of length differences between the sides of the Great Pyramid manifests a virtually negligible percentage error of 0.03%. Should these minuscule errors be our gold standard for investigations into the intent of the ancient builders? A reasonable compromise would be to set our benchmark for percentage error at the midpoint of these two figures, a minuscule 0.065%.

Application to Iconic Ratios: Towards a Standard of Analytical Rigor

Considering the above overview of the Giza Grid, we can now apply a rigorous standard of 0.065% for all measurements in this examination. These must meet or exceed our standard when aligned against iconic mathematical ratios such as the Golden Ratio (1.61803:1), the Golden Ratio Squared (2.61803:1), the √2 (1.41421:1 or 2.41421:1), and the √5 (2.23606:1). Therefore, with an acceptable error of 0.065% or less, the empirical comparison between any two connected Grid Lines should lie within a specific numerical range, which can be precisely calculated as:

The Golden Ratio = (**1.61697 - 1.61908**).
The Golden Ratio Squared = (**2.61633 - 2.61973**).
√2 = (**1.41329 - 1.41513**) or (**2.41264 - 2.41578**).
√5 = (**2.23461 - 2.23751**).

So, alongside the Giza Grid, *percentage error* serves as an indispensable tool for gauging the accuracy of our empirical analyses against established mathematical ideals. It provides a numerical lens through which we can scrutinize the reliability of our measurements. While one or two measurements with low percentage errors could be dismissed as coincidental, a suite of ratios with miniscule errors could signify a compelling approximation of the truth. Therefore, in

the context of the Egyptian enigma that we seek to decode—melding mythological beliefs with mathematical elegance—this analytical approach may illuminate a path towards a more integrated understanding of this ancient civilization.

The Sacred Point

In the blending of Egyptology and its interface with mathematical structures, one primordial point of convergence on the novel Giza Grid merits particular attention: point B5. The hypothesis of this paper posits that B5 acquired a sacrosanct status either during planning or immediately after the construction of Khufu's pyramid. Situated exactly on the axes and junction of both Khufu and Khafre's capstones, point B5 ostensibly served as an observational locus for the midwinter sunrise—a celestial event imbued with profound symbolism in Heliopolitan cosmology (Figure 12).

Khafre, Khufu's successor, appears to have utilized the 'sacred' point B5 in a manner that far surpasses mere astronomical observation. Khafre used the location of B5 to precisely center the Easterly axis of his own pyramid. Then the measure from B5 to B7 (218.90 m) was simply 'squared off' by the golden ratio (354.40 m) to precisely center his pyramid's Northerly axis. This makes sense, for the golden ratio was foundational in both the architecture of Khufu's pyramid and Hemiunu's serdab as will be shown.

The conceptual unification of Khafre's pyramid with that of Khufu seems to have been an underlying objective, best illustrated by the intended placement of Khafre's capstone at point E5. Such a strategic alignment would not only conceptually tether the two pyramids through the golden ratio, but it would also 'permanently' sanctify point B5 as a midwinter observation point. This confluence had esoteric significance for both the observer and the entombed pharaohs. It directed the observer to position himself at the geometrically significant right-angled junction of B5, where the two capstones and thus the two pharaohs meet. In doing so, an observer positioned at B5 could serve a metaphysical function akin to a Statue of the Ka found in the serdab of mastabas of the period, enabling the deceased pharaohs to vicariously experience the rising of Atum-Ra through the eyes of the living, proxy observer. In this way point B5 was for the pyramid complex like the serdab was for the Mastaba's serdab. It was a location from where the living observer could extend a form of corporeal agency to the deceased pharaoh annually.

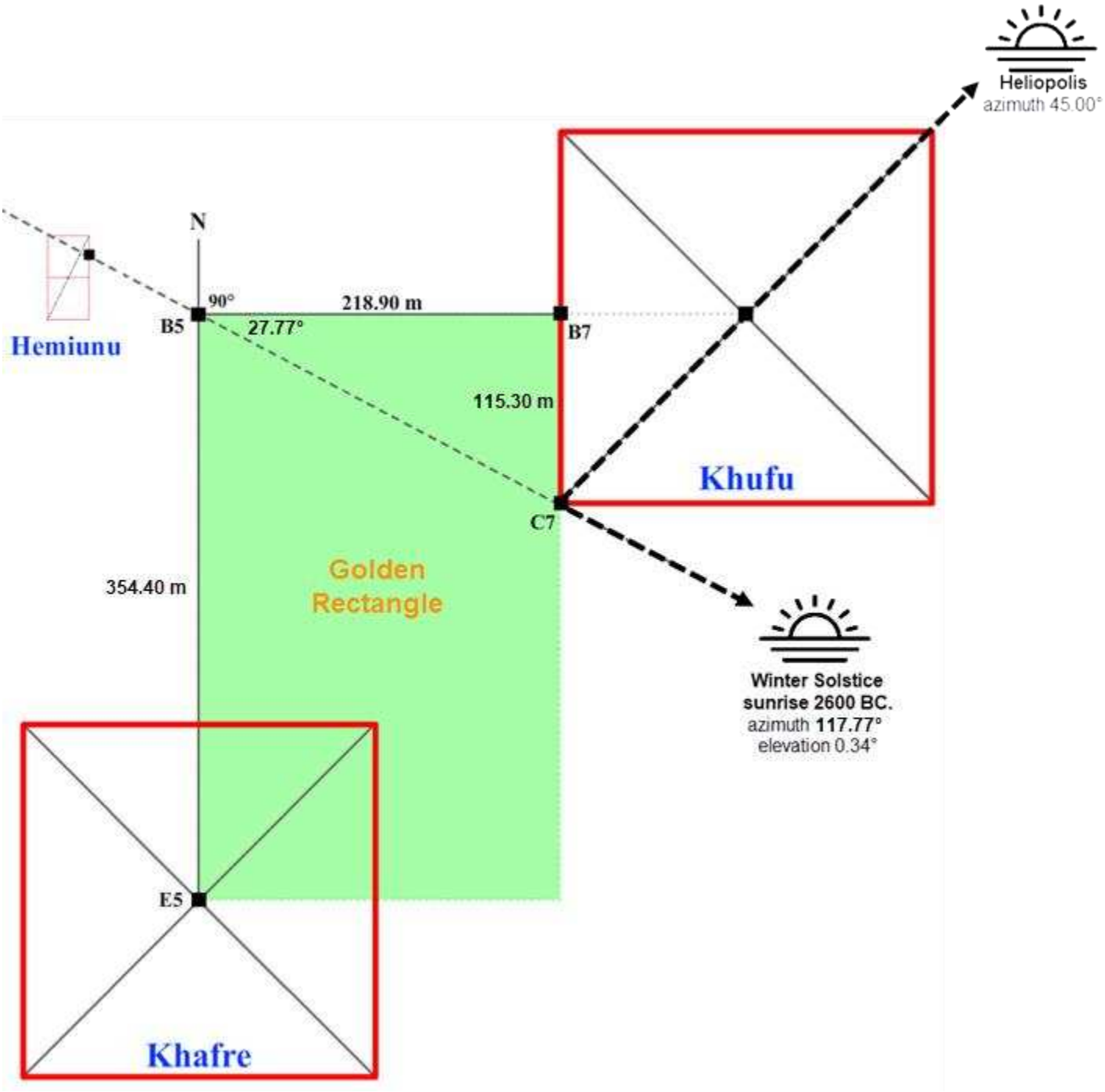


Figure 12 – Khufu, Khafre & the golden ratio solidify B5 to sight the winter solstice over C7.

Geometric Insights and Astronomical Alignments in Hemiunu's Mastaba

The study of ancient Egyptian mastabas provides a remarkable window into the intricacies of funerary practices and societal structures during the early dynastic period. One mastaba, attributed to Hemiunu, a priestly figure recognized for his role as the architect of the Great Pyramid of Giza and his high-ranking status in Pharaoh Khufu's reign, offers a wealth of insights into its dimensions, layout, and enigmatic symbolism. This section explores the intersection of Egyptian mathematics, funerary rituals, and architecture to unravel the significance of key measurements, including the fascinating relationship between the √5, the Ka statue, the azimuth, and the "Mastaba's Eye."

The term "mastaba," derived from the Arabic word "mastabah" meaning bench, refers to ancient Egyptian tombs that proliferated during the early dynastic period. These rectangular structures with flat roofs, constructed from mud-bricks or stone, were used as elevated burial sites for the elite and their families. The standard ground plan of a mastaba encompassed an offering chapel, a serdab (a concealed chamber housing a statue of the deceased), and a subterranean burial chamber accessible via a vertical shaft. As precursors to the iconic pyramids, mastabas are a pivotal link between rudimentary pit graves and the more elaborate pyramid complexes.

Situated within Giza's West Field necropolis and identified as G4000, Hemiunu's mastaba (Figure 13) stands as a testament to architectural magnificence during the Fourth Dynasty. Hemiunu's familial connection to Pharaoh Khufu and the prominent titles of Vizier, Priest and Overseer of All Construction he held underscore his significance. The core structure of his mastaba employed limestone blocks, while the exterior featured fine Tura limestone casing, exemplifying meticulous craftsmanship and reverence. Beneath the structure lies a subterranean burial chamber accessed via a vertical shaft of considerable depth.

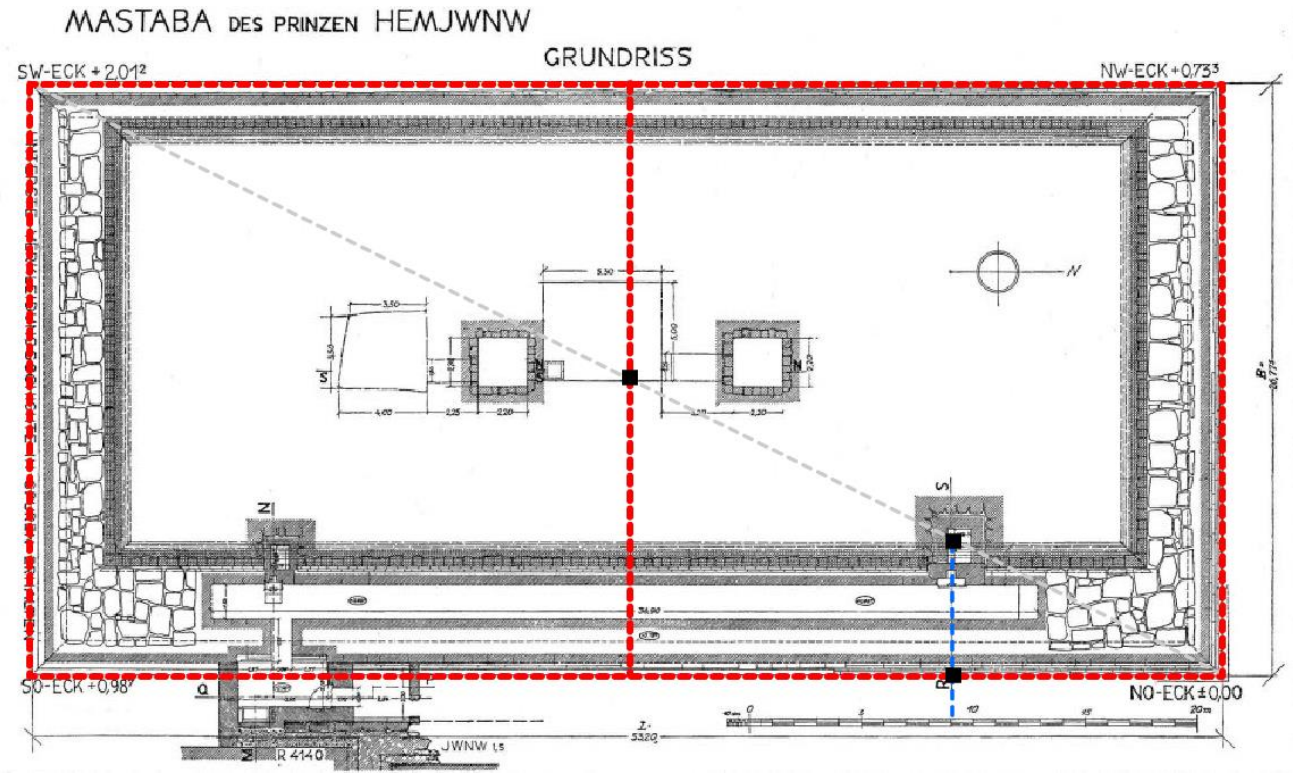


Figure 13 – Hemiunu's mastaba as a 2:1 ratio (double square).

Hemiunu's mastaba presents a compelling case study in the juxtaposition of mathematics and symbolism. The narrow offering chapel, housing niches to the north and south, unveils a unique arrangement, with the north niche dedicated to offerings intended for Hemiunu's Ka statue, enshrined within the adjoining serdab. The location of the serdab may underscore Hemiunu's integration of mathematical proportions within his preferred ritualistic practices, aimed at

honoring and sustaining the deceased in the afterlife. The once concealed serdab situated behind the northern niche, housed a limestone Ka statue that epitomizes the finesse of Old Kingdom sculpture. Accessible solely through a narrow slit or aperture within the northern niche—a feature aptly named the "Mastaba's Eye"— Hemiunu's design reflects the age-old Egyptian conviction that the Ka statue must possess the ability to "see" and "smell" offerings while remaining sheltered.

The dimensions of Hemiunu's mastaba are not mere measurements but an embodiment of mathematical relationships. Drawing from Hermann Junker's meticulous survey, the mastaba's foundational dimensions of 53.20 meters in length and 26.77 meters in width reflect the profound significance of the double-square and the √5. By utilizing Google Earth Pro, the positioning of the mastaba's northeast cornerstone emerges at 514.85 meters from the eastern base of Khufu's pyramid. This too very closely approximates the √5, evident through the ratio 514.85 / 230.40 = 2.2346, with a mere 0.065% deviation. Remarkably, this relationship imbues the side of the Great Pyramid (230.40 meters) with the value of 1, while casting the distance between the egress point of Hemiunu's eastern visage and the western base of Khufu's pyramid as principally 1.2346. In fact, because Hemiunu's mastaba is built with a slope angle at 76° and his Ka statue had an eastern face egress point (point H) about 1.5 meters high, the exact measurement between the virtual egress point and the western face of Khufu's pyramid slightly exceeds 515 meters, yielding perhaps an exact 1.236 : 1. The intricate symmetry evident in these measurements not only evokes an understanding of the √5 building blocks of the golden ratio, but, as we will see, also mirrors the placement of Hemiunu's serdab within his mastaba (Figure 14).

Applying the measurement data furnished by Hermann Junker, we can discern the precise placement of Hemiunu's serdab (point E), whose aperture is often referred to as the "eye" of the mastaba, situated at 11.89 meters south of the mastaba's northern face. Envisioning Hemiunu's statue gazing eastward from his serdab through this symbolic ocular orifice, a captivating revelation emerges: his line of vision would create an egress point (point H) on the eastern facade of his mastaba. It is vital to recognize that the entire mastaba served as a corporeal representation of Hemiunu, underscoring that the point at which his line of sight exits the structure, in accordance with the golden ratio, essentially becomes a conduit of vision for his Ka Statue to perceive the world.

The Golden Ratio is a mathematical construct that possesses an intriguing allure. Contrary to some interpretations, it is not merely a singular number but represents a ratio derived from an irrational number juxtaposed with unity (the number 1). This ratio can be approximated by the irrational 1.61803 : 1. However, from a rational standpoint, it can also be represented by the ratio 233 : 144, 89 : 55, 34 : 21, or even 13 : 8, indicative of its connection to the Fibonacci sequence.

The distinguishable intrigue surrounding the Golden Ratio dates to at least the era of Euclid. Historically, its irrational character was discovered to be rooted in the geometry of a double-square, which is a rectangle with a 2:1 aspect ratio. Specifically, the foundation lies in the comparison of the sides of the double-square to its diagonal, which has a length of

√5. To elucidate, when one adds the diagonal length of a double-square to its shorter side of 1 unit, and then divides by its longer side of 2 units, the Golden Ratio emerges: $\frac{\sqrt{5}+1}{2} = 1.618.$ However, an 'inverse' geometric expression can be derived which is arguably more elegant: $\frac{2}{\sqrt{5}-1} = 1.618.$

Instead of venturing outside the sacred parameters of the double-square, this inverse construction, which shall be referred to as Figure 14 for illustrative purposes, adroitly pivots inward. By intersecting its diagonal of length √5 (denoted as segment C-B) with the length of its side (D-B) at a particular point, say E, two distinct segments materialize. The measurements of these segments are 1.00 units (for E-B) and 1.236 units (for E-C). As a result, when one compares the length A-B (2.00 units) to the segment E-C (√5-1), we observe the manifestation of the Golden Ratio, as delineated previously.

Intriguingly, upon extending point E in both vertical and horizontal dimensions, four additional ratios emerge, specifically EF:EG, EH:EG, EF:EH, and BD:DF. A mathematical analysis reveals their significance: EF:EG approximates 1.618 : 1 (reiterating the Golden Ratio), EG:EH resonates with 2.236 : 1, EF:EH offers 1.236 : 1, and BD:DF yields 1.118 : 1.

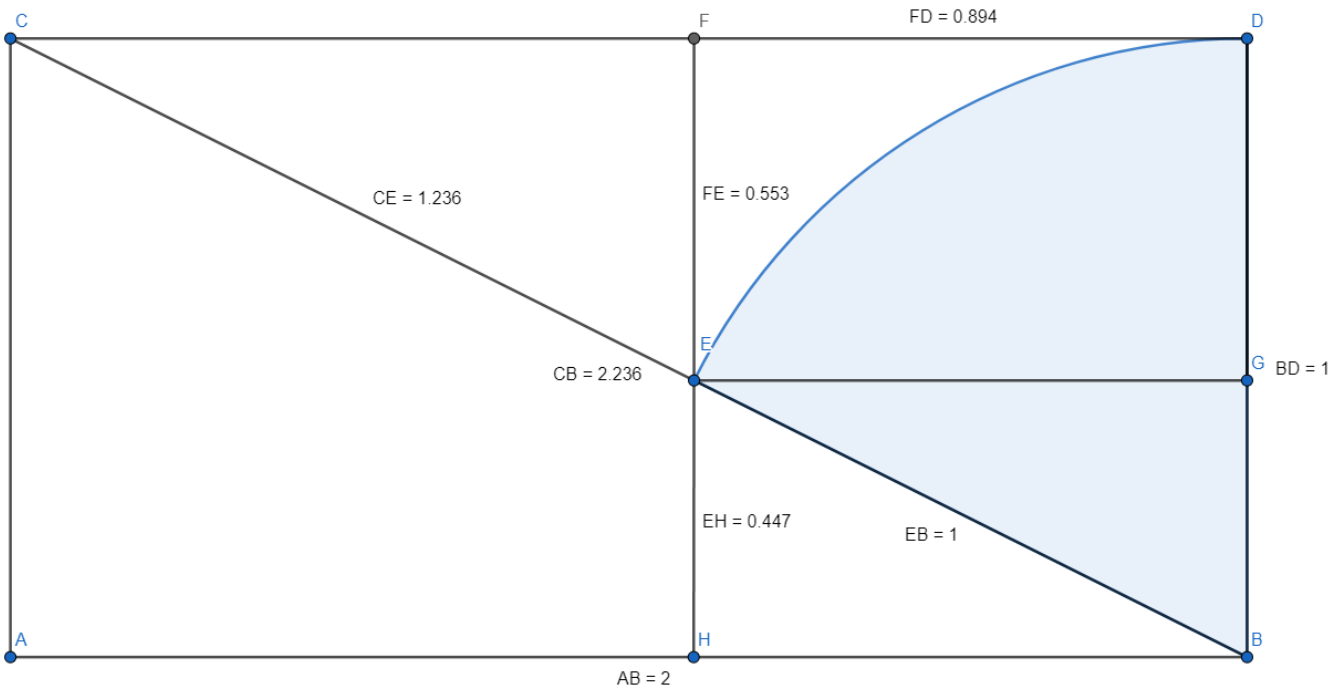


Figure 14 – Topology and positioning of Hemiunu's serdab (E) in accordance with the √5 & golden ratio.

In the grand survey of ancient Egyptian architecture and design, the importance of Hemiunu's geometric structure is not alone and the golden ratio in its inherent mathematical beauty. True profundity emerges when one recognizes that corresponding ratios can be identified within the majestic edifice of Khufu's pyramid and its surroundings on the Giza Plateau. Hemiunu, now revealed to have a distinct fascination with the √5 and the golden ratio, meticulously positioned his Ka statue within his mastaba at the essential point E. Line EH signifies the Ka's sightline egressing from the statue,

through the niche, to the eastern face of his mastaba, offering a unique window through which his Ka is believed to 'perceive the world'. Therefore, a careful superimposition of Figure 14 atop Junker's ground plan unveils that Hemiunu likely intended to segment the diagonal of his mastaba at point E to not only strategically position his Ka statue in accordance with the Golden Ratio, but also to give his Ka the ability to observe the Winter Solstice sunrise (Figure 15). In executing this design, Hemiunu could have certainly envisioned line EH as a conduit of vision leading from his statue, through the niche, to the eastern facade of his mastaba. Amazingly, a line drawn from the Southwest cornerstone (C7) of Khufu's pyramid, through point B5 on the Giza Grid, terminates not only on Hemiunu's mastaba, but definitively on point H (Figure 12). This alignment would orchestrate a breathtaking spectacle for Hemiunu's Ka: a 117.77° direct view of Ra's Winter Solstice sunrise, majestically ascending over the cornerstone of the resurrected Khufu's pyramid.

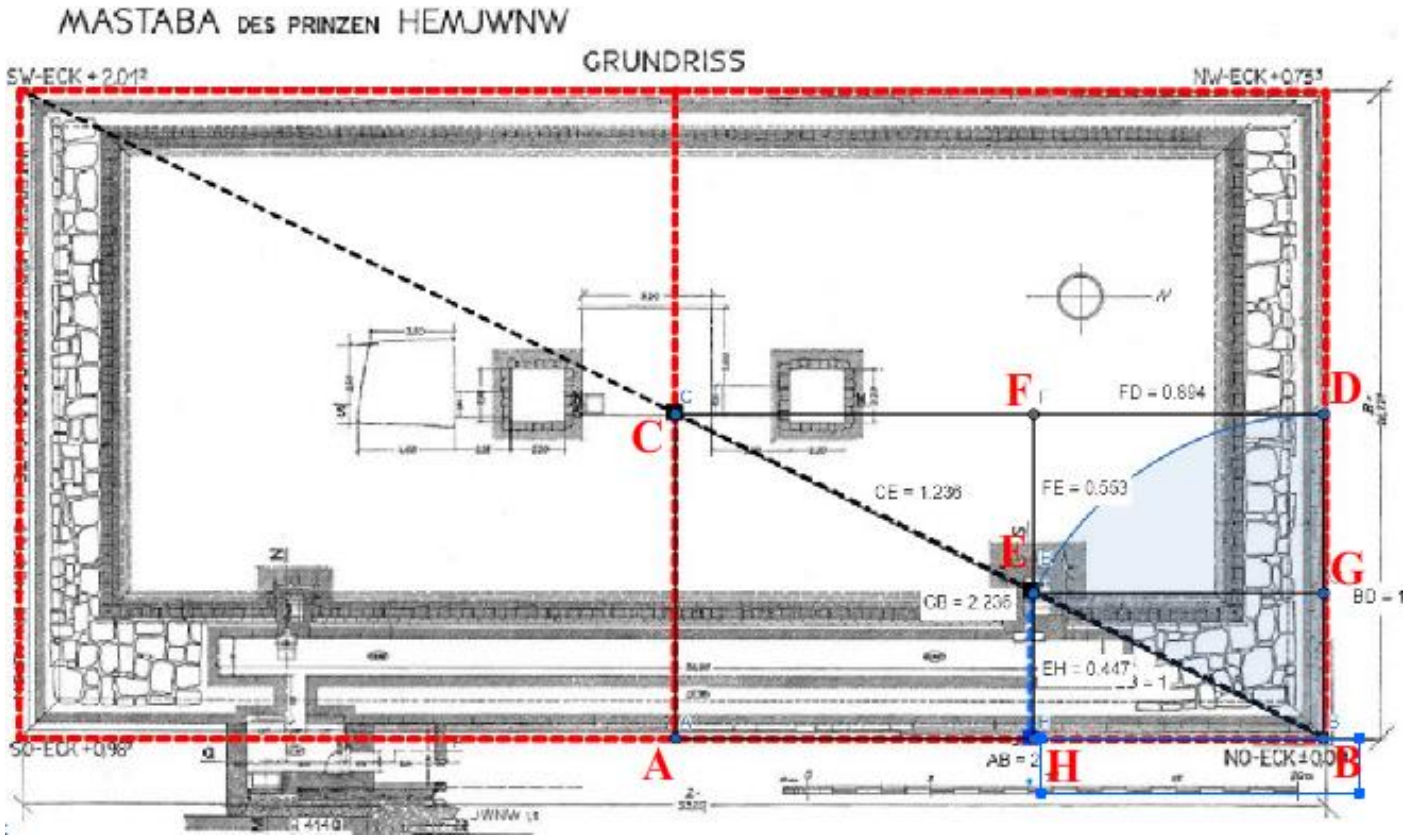


Figure 15 – Overview of Hemiunu's mastaba & serdab in accordance with the golden ratio and √5.

Using the Giza Grid to observe the Winter Solstice sunrise over Khufu's cornerstone, ca 2600 BC.

From Khufu's cornerstone C7, create the green rectangle C7-B7 : B7-B5 which equals 115.30 : 218.90 meters (Figure 16). The Northerly measure (C7-B7) of this rectangle is calculated by subtracting the 'north' coordinate of Khufu cornerstone C7 (N 99884.7) from Khufu capstone B8 (N 100000), resulting in a measure of 115.30 meters. The Easterly measure (B7-B5) of this rectangle is calculated by subtracting the 'east' coordinate of Khafre's capstone E5 (E 499666.00) from Khufu

cornerstone C7 (E 499884.90), resulting in a measure of 218.90 meters. The hypotenuse (B5-C7) of this rectangle generates an angle measuring 27.77° which creates a line-of-sight azimuth of 117.77° (90° + 27.77°) to the horizon as viewed over cornerstone C7. Why is this relevant?

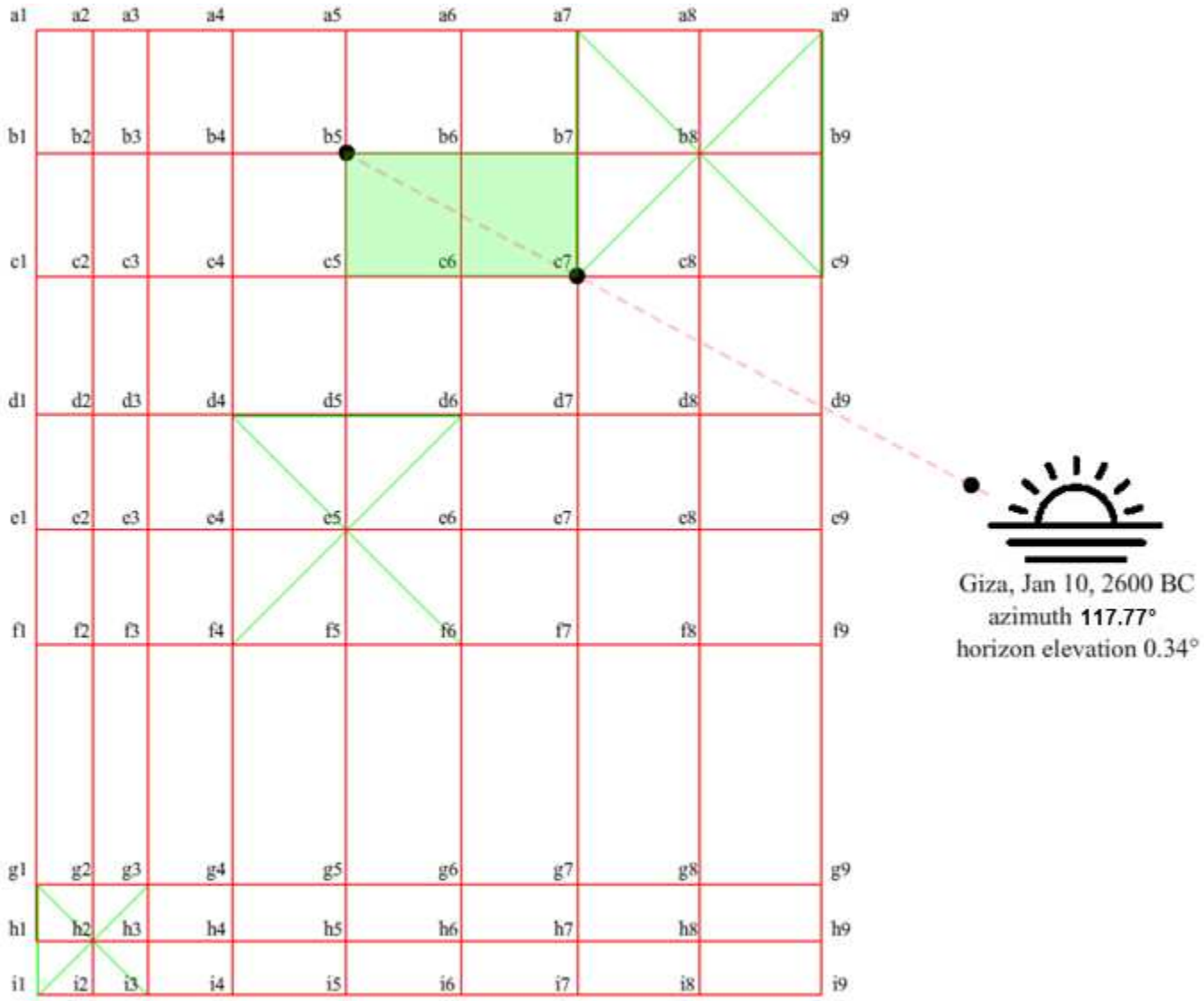


Figure 16 – The computational Giza Grid in accordance with Dash's 2016 survey.

In 2600 BC the midwinter sunrise at Giza occurred on January 10th. The elevation of point B5 on the Giza plateau is 73 meters above sea-level and the elevation of the distant hills (36 miles away at an azimuth of 117.77°) is 457 meters, creating a net-elevation horizon of 384 meters, or +0.34°. The Starry Night program automatically adjusts for the sun's angular refraction and the earth's obliquity (ca 2600 BC), but the adjustment of the horizon needs to be done manually. So, after manually raising the elevation of the SE horizon by +0.34°, it can be observed that the upper-limb of the midwinter sunrise at Giza in 2600 BC occurred at 7:02:28 AM at an azimuth of 117.70°. This azimuth is remarkably close to the B5-C7 hypotenuse used as a line-of-sight from the Sacred Point to cornerstone C7 at an azimuth of 117.77°. In fact, it is only a miniscule difference of 0.07°, or a 'sighting' percentage error of only 0.059%. Amazingly, the sun would be

exactly 117.77° on the horizon only a mere 36 seconds later at 7:03:04 AM with two-thirds of its disc still below the horizon as demonstrated in Figure 17.

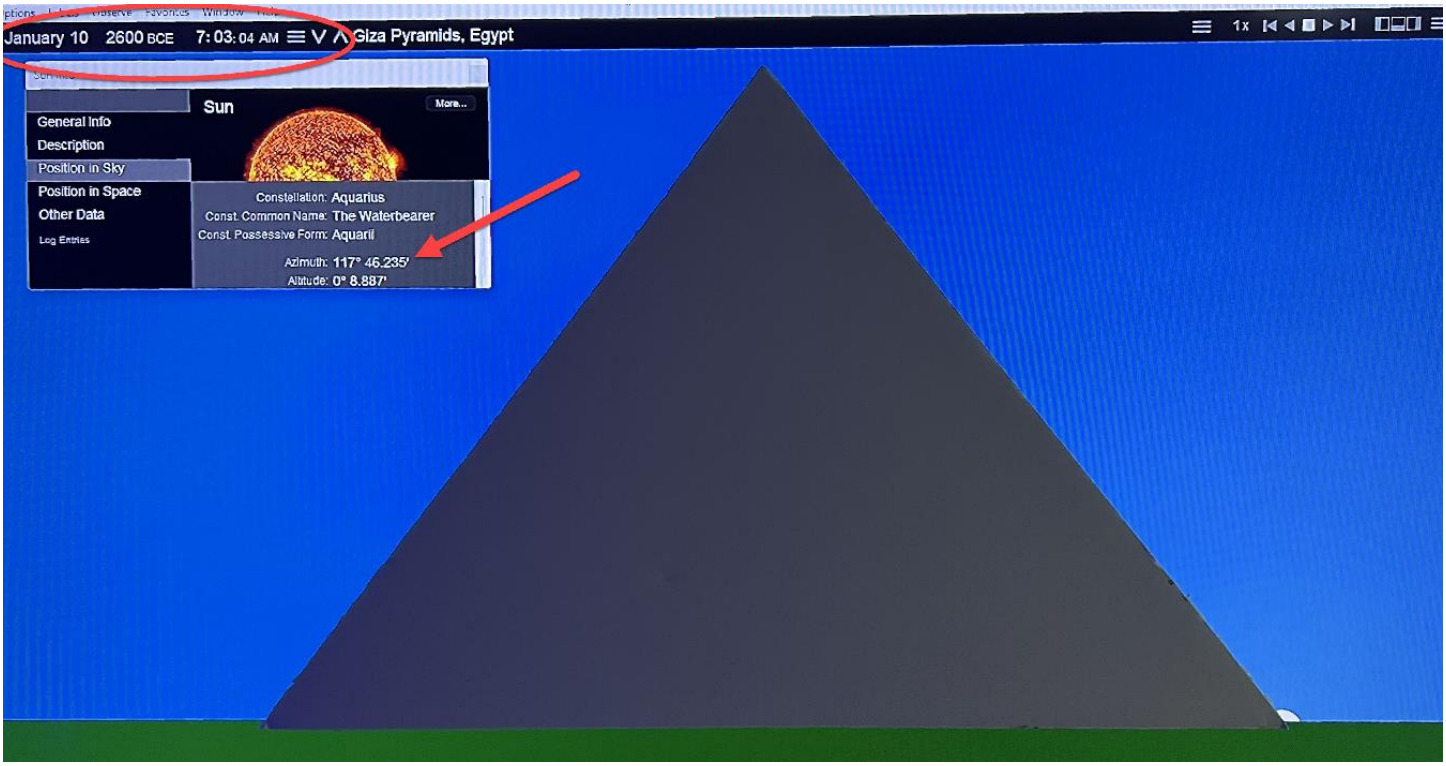


Figure 17 – Winter solstice sunrise over cornerstone C7 as would be seen from sacred point B5 (ca 2600 BC).

Therefore, we see that point B5 -- which is centered on both Khufu's and Khafre's capstone -- was a primary 'sacred point' from which to view the midwinter sunrise over the cornerstone C7 of Khufu's pyramid. Because B5 was 'sacred', Hemiunu likely planned to center the next pyramid's (Khafre's) capstone on E5 by squaring-off the distance B5-B7 in accordance with the golden ratio. Once it was determined that Khafre's capstone was to be placed on E5, it next became a question of how large to make the base of Khafre's pyramid by calculating its apothem, E5-E4. For this, another ratio relative to Khufu's pyramid was made. It was a ratio based upon the preestablished measure of the Remen : Royal Cubit or 1: √2; what is now known as the silver ratio. By simply extending the side A9-A7 of Khufu's pyramid by √2 (which equals the length of Khufu's diagonal which points to Heliopolis) to point A4, Khafre simultaneously defined both the western extreme and apothem length (E5-E4) of his own pyramid. From there it was only a matter of squaring everything off. See the blue √2 rectangle of Figure 19.

Once Khafre's pyramid was built it was then determined to center – in accordance with the golden ratio -- Menkaure's capstone in geometric 'alignment' with both his father's and his grandfather's capstones. See the yellow golden ratio rectangle in Figure 20. Menkaure defined the North-South axis of his capstone by first positioning himself on E8, which,

being opposite sacred point B5, was the center junction of Khufu and Khafre's capstones.[7] He knew that the measure B8-E8 was the same as B5-E5 and in accordance to the golden ratio, so he used that measure and headed directly west, through his father's capstone (E5) to point E2, in order to create a larger golden rectangle E2-B2 : E2-B8 (1.6182 : 1) which not only defined the Easterly axis (capstone) of his own pyramid, but amazingly squared his father's rectangle in perfect proportion to the golden ratio. Meaning that the long measure of Khafre's 'grey' golden rectangle in Figure 18 became the short measure of Menkaure's yellow Golden Rectangle in Figure 20. Or said in another way, the ground distance between Khufu and Khafre's capstones (B8-E8) is the short side of the yellow golden rectangle and the ground distance between Khufu and Menkaure's capstones (E2-E8) is the long side of yellow golden rectangle. In this way – like the later segment AC in Euclid's Book 13, proposition 5[8] -- the distance of B5-B7 was used as a foundational sacred distance, expressed mathematically as:

B5-B7 = 1.000

B8-E8 = 1.618

E2-E8 = 2.618

Now that Menkaure had the North-South axis of his capstone defined via the golden ratio, he then needed to determine both the apothem (size) of his pyramid and its East-West axis. It is difficult to fathom that he would not once again use the golden ratio in his plan. So, by centering himself again on E8 – the secondary junction of his ancestors' capstones -- he then multiplied the measure between E8 to the western face of Khufu's pyramid (E8 – C8) by the golden ratio squared (2.6185 : 1) and again headed directly west through his father's capstone (E5) to point E1, thus creating the apothem length (E1-E2) and western limit of his own pyramid. See the pink rectangle of Figure 21, which again can be observed in Euclid Book 13, proposition 5.[9] What makes this even more amazing is that by positioning Menkaure's pyramid in its precise location, not only does the whole pyramid plan of Giza demonstrate the use of the Golden Ratio and the √2, but, as demonstrated by John Legon, it also mimics the celestial layout of Orion's belt by synthesizing the whole grid into a precise √2, √3, √5 triangle (Figure 22).[10] For the distance A1-A9 is 741.50 meters and the distance A1-I1 is 908.00, resulting (according to the Pythagorean Theorem) in the distance A9-I1 to be 1172.30 meters. Therefore, if A1-A9 is to represent √2, then A1-I1 and A9-I1 must represent √3 and √5 respectively, for:

√3 / √2 = 1.224 = 908.00 / 741.50

√5 / √2 = 1.581 = 1172.30 / 741.50

Astounding!

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[7] There are only six of these junctions on the Giza Plateau, B5, E8, B2, H8, E2, H5.

[8] Euclid's rectangle AB-BK compared to DB-BE is in the same proportion as the rectangles found in Figures 17 and 19.

[9] Euclid's rectangle DB-BK is in the same proportion as the rectangle found in Figure 20.

[10] John Legon, 'Plan of the Giza Pyramids'. Legon is the first to recognize the near perfect use or irrationals at Giza. Also, in 2016 Gary Meisner discovered a tight tolerance rectangle what I now call the B8-E8 golden rectangle, see http://www.goldennumber.net. For a presentation on both the Giza plan and the Orion Correlation Theory see Bauval & Glibert 'The Orion Mystery'.

The positional arithmetic of Khafre's Golden Ratio (Figure 18)

From point B5, create ratio B5-B7 : B5-E5 which equals 218.90 : 354:40 meters and is in accordance with the golden ratio (Fig 17). The North-South measure B5-E5 of this ratio is calculated by subtracting the 'north' coordinate of Khafre's capstone (N 99645.60) from the 'north' coordinate of Khufu's capstone (N 100000.0), resulting in a measure of 354.40 meters. The East-West measure (B5-B7) is already determined as above, resulting in a measure of 218.90 meters. From these measurements the rectangle B5, E5, E7, B7 can be created.

Khafre's first ratio is an extremely close approximation of the Golden Ratio for 354.40 / 218.90 = 1.6190, an error of only 0.059%. As stated above, it was likely planned that the capstone of Khafre's pyramid should be placed at E5 to conceptually unify Khafre's pyramid with Khufu's pyramid through the golden ratio and the 'sacred' midwinter observation point of B5. This would have the added benefit for the observer and both deceased pharaohs, who, by directing the observer to stand precisely at the right-angled junction of their two capstones at B5, could, like a Statue of the Ka, use the eyes of the bodily observer to behold the rising of Ra, and as a proxy give life to their deceased eyes.

The positional arithmetic of Khafre's silver ratio (Figure 19)

From point A9, create the ratio A9-A7 : A7-A4 which equals 230.40 : 326.00 meters and is in accordance with the silver ratio (Fig 18). A9-A7, the first East-West measure of this silver ratio, is calculated by subtracting the 'east' coordinate of Khufu's A7 cornerstone from Khufu's A9 cornerstone, resulting in a measure of 230.40 meters. A7-A4, the second East-West measure of this silver ratio, is calculated by subtracting the 'east' coordinate of Khafre's F4 cornerstone from Khufu's A7 cornerstone, resulting in a measure of 326.00 meters. From these measurements the rectangle A9, A4, C4, C9 can be created. Khafre's second ratio is an extremely close approximation to the Silver Ratio for 326.00 / 230.40 = 1.4149, an error of only 0.050%.

The positional arithmetic of Mekaure's first golden ratio (Figure 20)

From point B8, create the ratio B8-E8 : B8-B2 which equals 573.50 : 354.40 meters and is in accordance with both the golden ratio and the golden ratio squared (Figures 17, 19 and 20). The North-South measure (B8-E8) of this ratio is taken from the same North-South measure of Khafre's first golden ratio described above and is calculated by subtracting the 'north' coordinate of Khafre's capstone (N 99645.60) from the north coordinate of Khufu's capstone (N 100000), resulting in a measure of 354.40 meters. The East-West measure (B8-B2) of this ratio is calculated by subtracting the east coordinate of Menkaure's capstone (E 499426.50) from the east coordinate of Khufu's capstone (E 500000.0), resulting in a measure of 573.50 meters. From these measurements the rectangle B8, E8, E2, B2 can be created. Stunningly, Menkaure's first ratio is a near perfect approximation to the Golden Ratio, for 573.50 / 354.40 = 1.6182, an error of 0.011%.

The positional arithmetic of Menkaure's second golden ratio (Figure 21)

From point C8, create the ratio C8-E8 : E8-E1 which equals 626.10 : 239.10 meters and is in accordance with the golden ratio squared (Fig 20). The North-South measure (C8-E8) of this ratio is calculated by subtracting the north coordinate of Khafre's E5 capstone (N 99645.60) from the north coordinate of Khufu's C7 cornerstone (N 99884.90), resulting in a measure of 239.10 meters. The East-West measure (E8-E1) is calculated by subtracting the east coordinate of Menkaure's G1 cornerstone (E 499373.90) from Khufu's capstone (E 500000.0), resulting in a measure of 626.10 meters. From these measurements the rectangle C8-E8-E1-C1 can be created. This second golden ratio of Menkaure's is an extremely close approximation of the golden ratio squared for 626.10 / 239.10 = 2.6185, an error of only 0.02%.

Conclusion

As we delve deeply into the geometrically rich and spiritually laden architectural feats of the Giza Plateau, it becomes apparent that these edifices are much more than mere tombs or monolithic stone structures. They are embodiments of an ancient cosmogony and solar obsrvation interlaced with a profound mathematical understanding, a confluence that unveils the profound depth of ancient Egyptian civilization.

From a prima facie standpoint, one might see the Great Pyramid as a mere geometric entity - a square with its diagonals intersecting at 45° angles. Yet, this simplistic observation belies the depth of its foundational design, which is intertwined with the Golden Ratio's exquisite precision. The choice of the Golden Ratio is not arbitrary. Its prevalence in natural phenomena and unique status in geometry makes it a pertinent choice to represent the harmony and order of the universe. This universal proportion finds its epitome on the Giza Plateau, particularly in the structures' alignment and emergence of the Great Pyramid's capstone, leading the observer's eye upwards to the heavens.

The Giza Grid, while a new analytical tool, might have served as an invaluable template for the ancients, guiding the placement and orientation of the Giza Plateau's structures. An essential juncture of this grid, point B5, offers an observer a privileged perspective of the midwinter sunrise, casting its first rays over the sacrosanct point C7. This alignment is more than a mere astronomical phenomenon; it is a spiritual manifestation, a harmonious confluence of the mortal and the cosmic. The Golden Ratio and √2 inclusion further accentuates the significance of this alignment, connecting Khufu, Khafre and Menkaure's pyramids in an esoteric bond that resonates not only with the mastaba of Khufu's chief architect, Hemiunu, but also Heliopolitan cosmogony.

Speaking of Heliopolitan beliefs, the Benben stone occupies a paramount role. As a symbol of creation, the Benben stone's importance parallels that of the pyramid's capstone or Benben, serving as the nexus between the terrestrial and

the celestial. The story of Atum, his emergence from the watery abyss, and the subsequent creation of the cosmos is crystallized in the pyramid's geometry. The progression from a square base and its diagonals, emerging into a capstone, mirrors the cosmological narrative of the Benben stone's emergence from the primordial waters, symbolizing creation, transformation, and rebirth.

The meticulous alignment and orientation of the Great Pyramid towards Heliopolis, the abode of Ra, further underscores the deep reverence the ancient Egyptians held for the resurrected sun god and his daily journey across the sky. This alignment isn't merely a tribute but a manifestation of their belief in the cyclical nature of life and death, of darkness and light. Khufu's anticipated resurrection, viewed through the lens of the sun's rebirth after the winter solstice, exemplifies this cyclical belief, underscoring the hope and faith of a civilization deeply in tune with nature's rhythms.

As we contemplate the enigmatic mastaba of Hemiunu, the genius architect behind the Great Pyramid, we find a confluence of deep spiritual intent and precise mathematical articulation. The incorporation of the Golden Ratio and √5 in the mastaba's design, particularly in the alignment of the "Mastaba's Eye" with the Winter Solstice sunrise over Khufu's pyramid, attests to Hemiunu's profound understanding of both geometry and the cosmos. This intersection of the earthly and the heavenly, anchored in the Golden Ratio, serves as a testament to the ancient Egyptians' unparalleled ability to seamlessly intertwine spirituality, cosmology, and mathematics.

In summation, the architectural marvels of the Giza Plateau, encompassing the majestic pyramids, the intricate Giza Grid, and the enigmatic mastabas, unveil a tapestry of profound mathematical precision, deep spiritual intent, and a keen understanding of the cosmos. The interplay of the Golden Ratio, the meticulously calculated alignment with Heliopolis, and the spiritual symbolism of the Benben stone paint a holistic picture of a civilization that understood and revered the universe's intricate rhythms. Through their architectural feats, the ancient Egyptians have bequeathed to posterity a legacy that stands as a testament to their intellectual prowess and spiritual depth, a legacy that continues to intrigue, inspire, and captivate the minds and hearts of seekers across epochs.

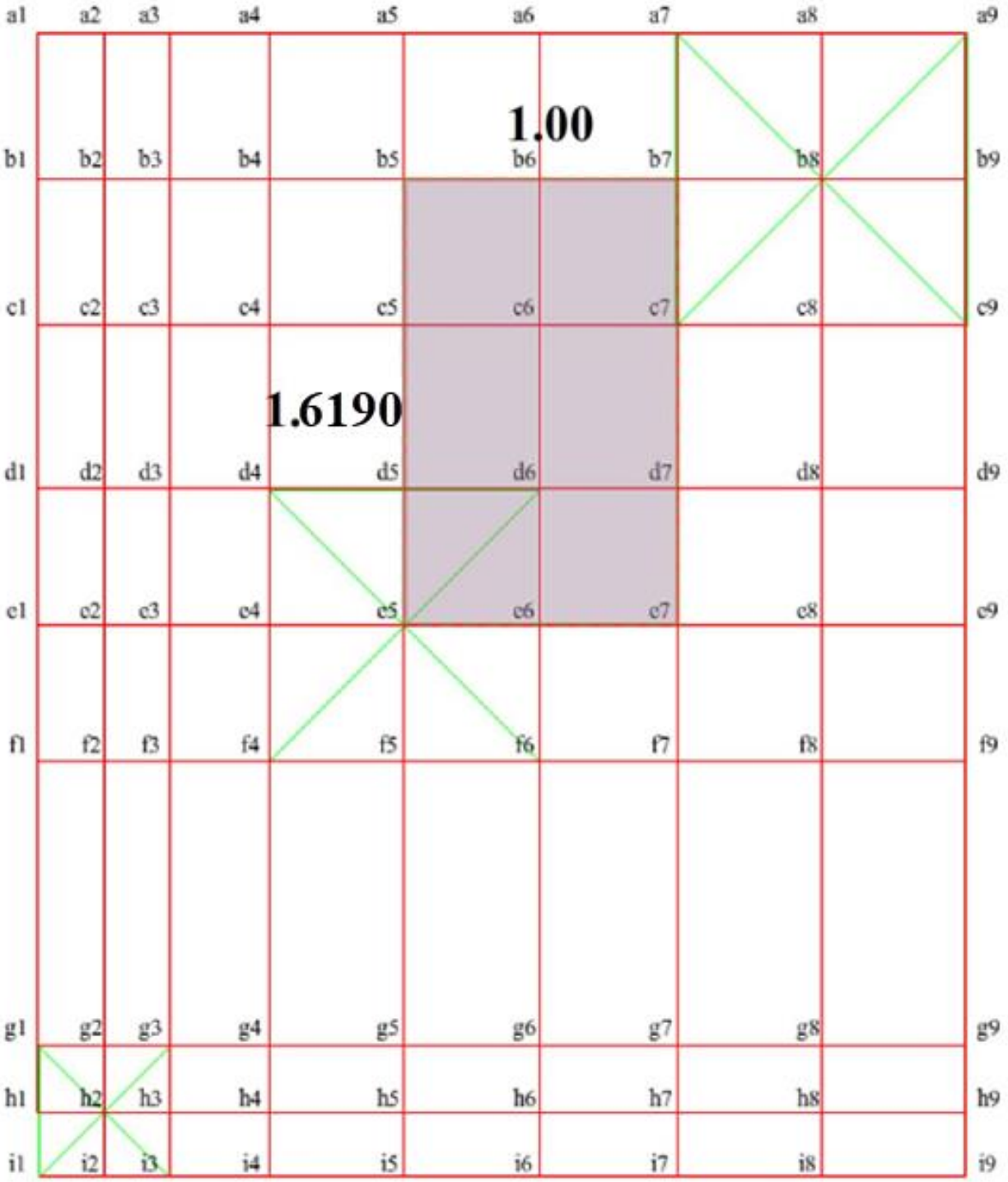


Figure 18 – Khafre's Golden Ratio

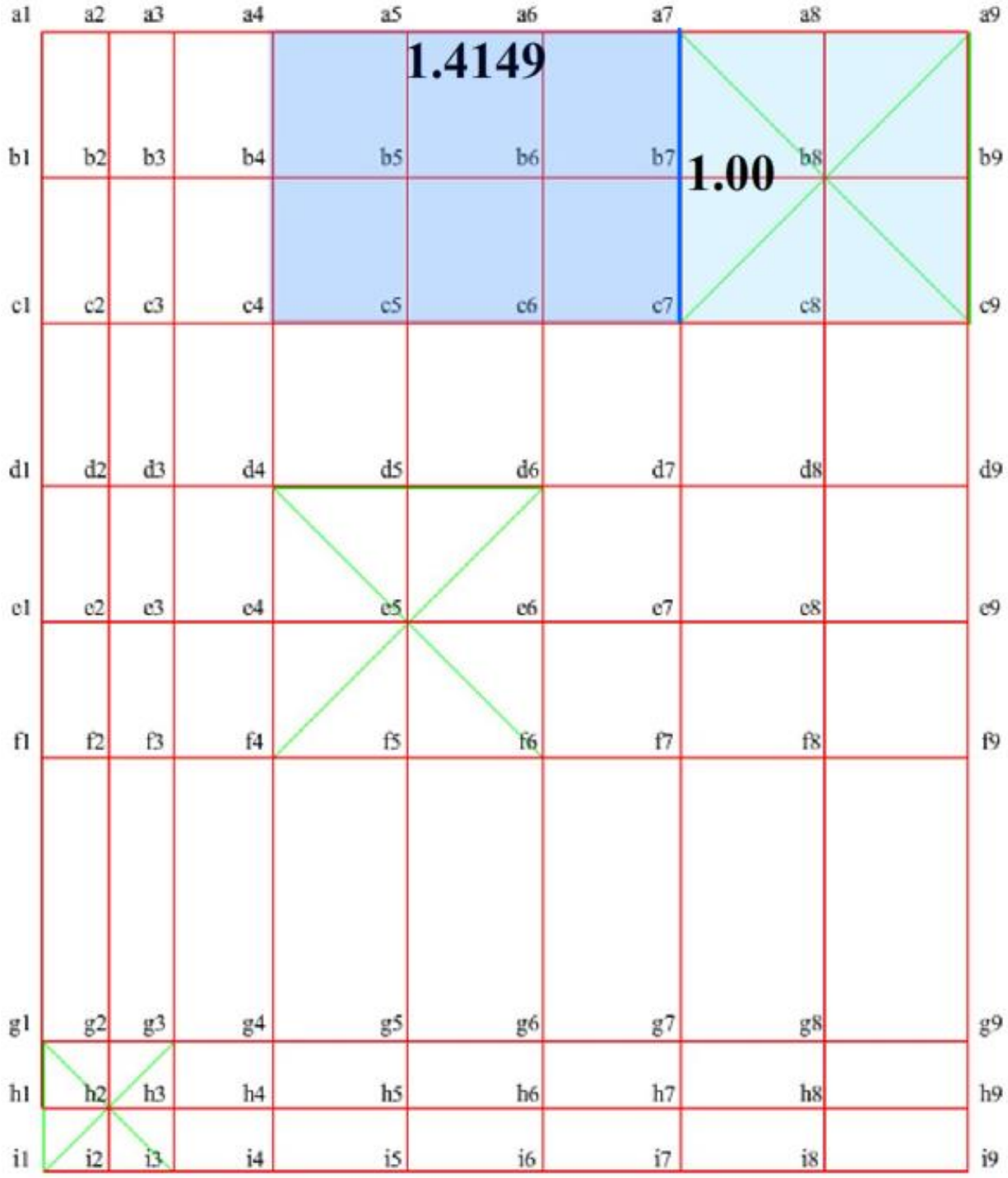


Figure 19 – Khafre's Silver Ratio

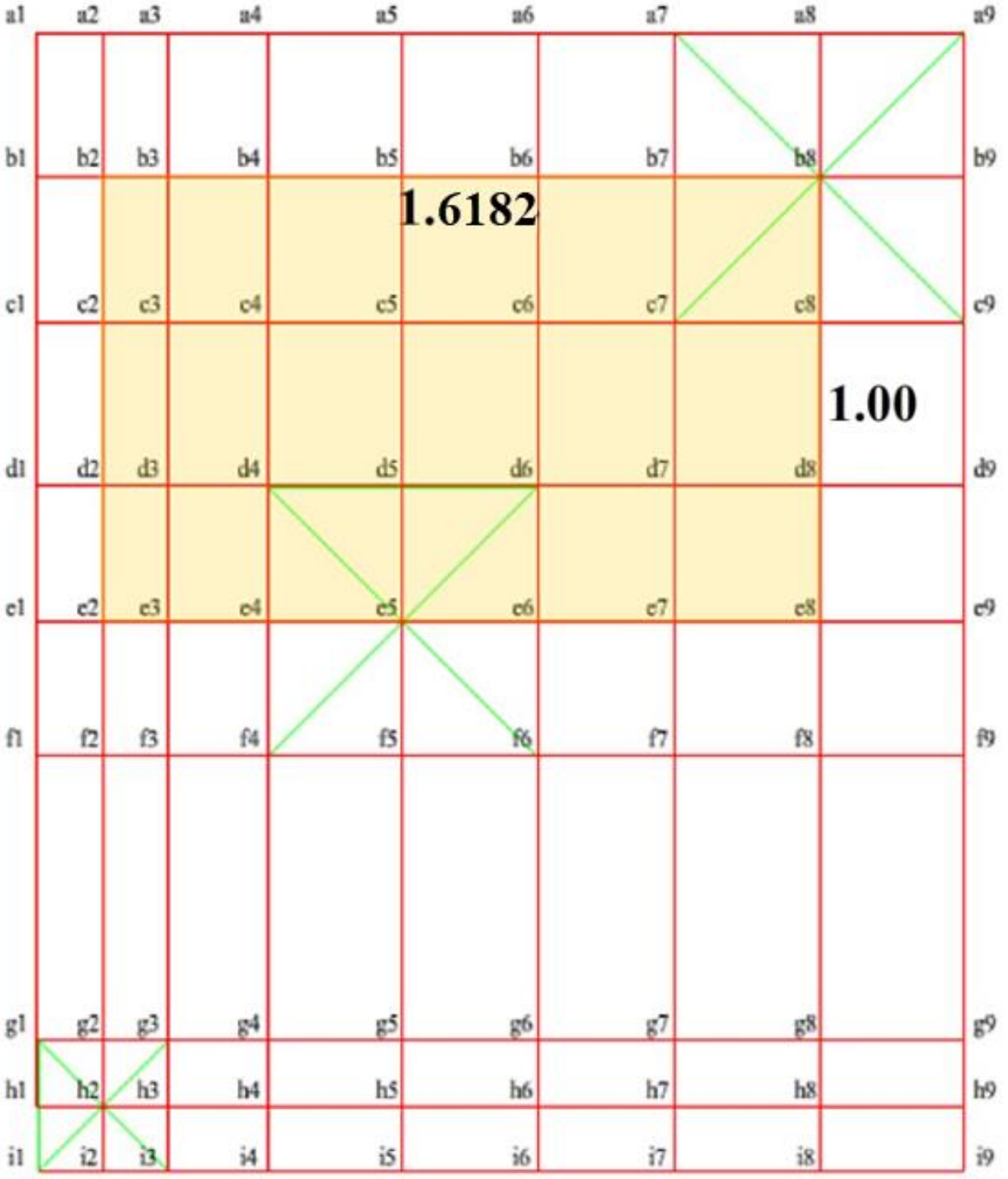


Figure 20 – Menkaure's First Golden Ratio

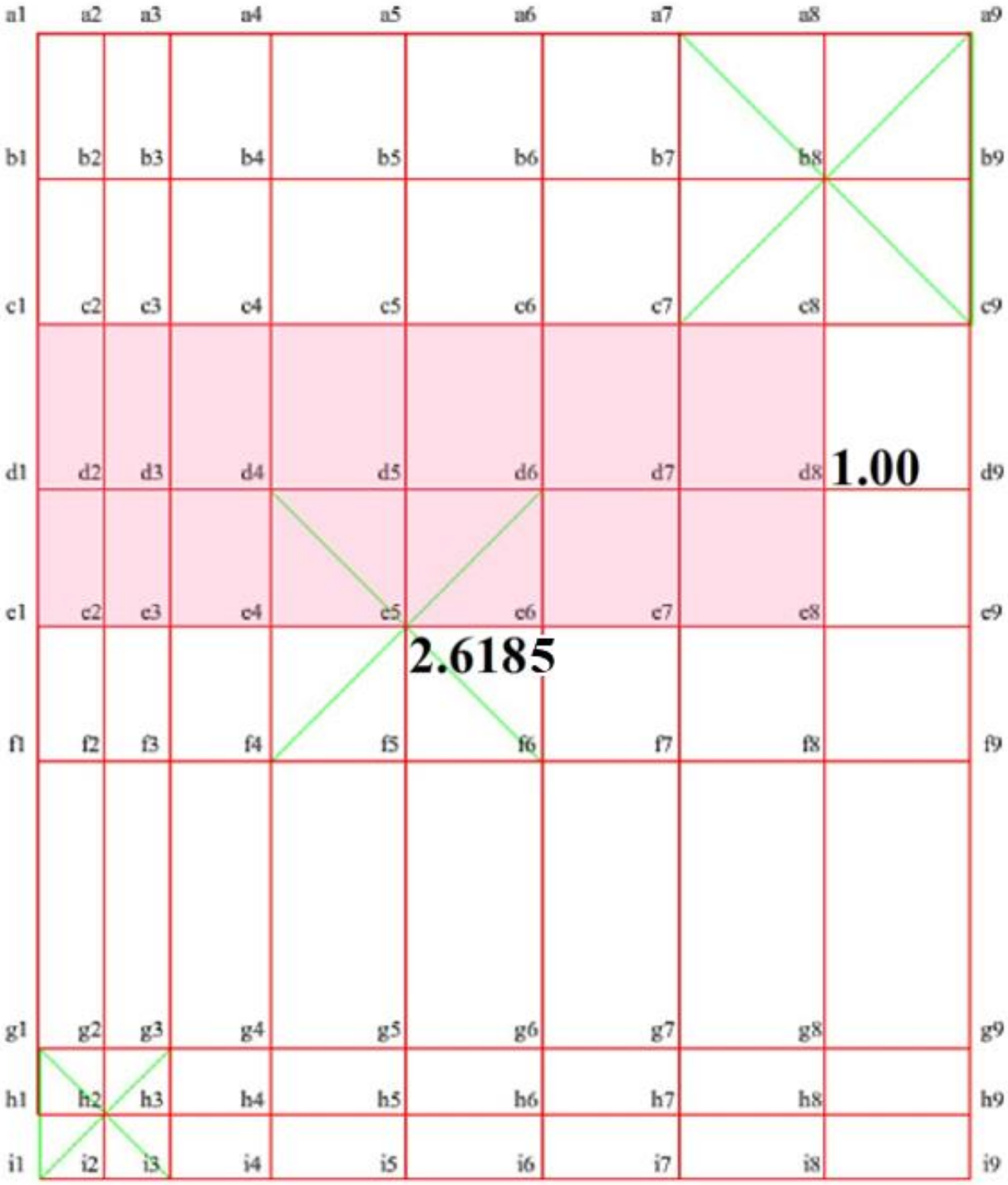


Figure 21 – Menkaure's Second Golden Ratio

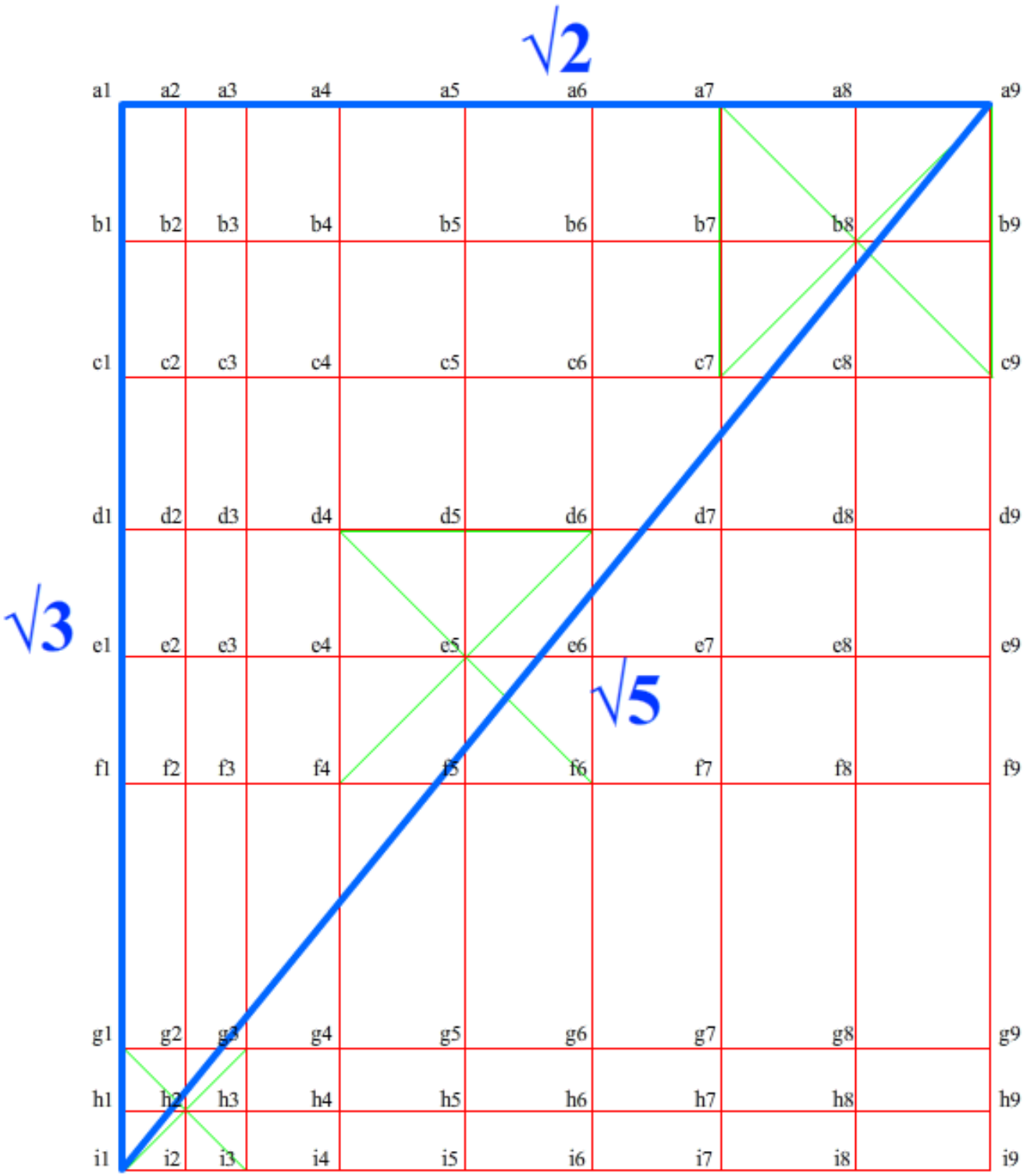


Figure 22 – The Grid's overall √2, √3 and √5 design

References

Dash, G. (2017). Where, Precisely, are the Three Pyramids of Giza?

Magli, G. (2020). Archaeoastronomy: Introduction to the Science of Stars and Stones. Germany: Springer International Publishing.

Magli, G. (2013). Architecture, Astronomy and Sacred Landscape in Ancient Egypt. United Kingdom: Cambridge University Press.

Belmonte, J. A., Lull, J., Mahoney, T. (2023). Astronomy of Ancient Egypt: A Cultural Perspective. Germany: Springer International Publishing.

Belmonte, J.. (2009). The Egyptian Civil Calendar: a Masterpiece to Organize the Cosmos.

Lehner, M. (2008). The Complete Pyramids. United Kingdom: Thames & Hudson.

Bauval, R., Gilbert, A. (2001). The Orion Mystery: Unlocking the Secrets of the Pyramids. United Kingdom: Penguin Random House.

Petrie, W. M. F. (1883). The Pyramids and Temples of Gizeh. United Kingdom: Field & Tuer.

The Thirteen Books of the Elements, Vol. 1. (1956). United Kingdom: Dover Publications.