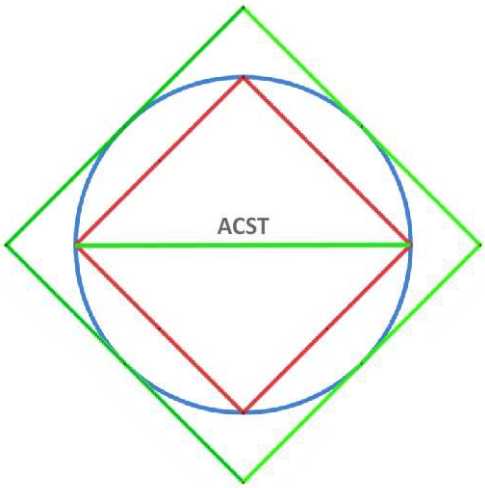
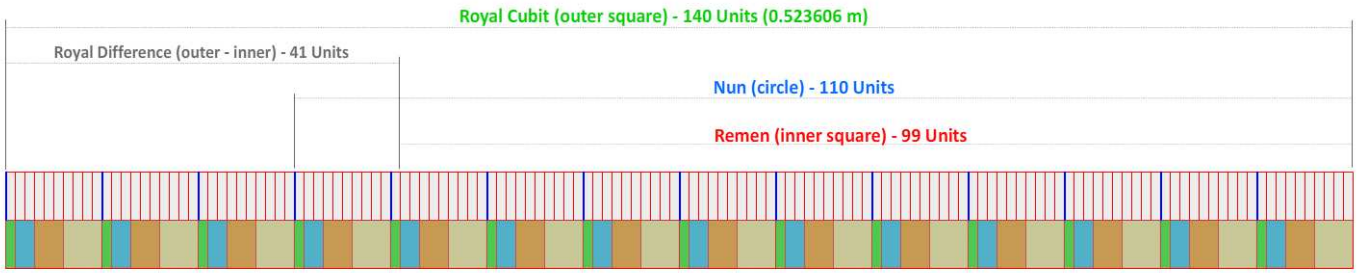




The ACST Cubit

HOW THE GEOMETRICAL RELATIONSHIP BETWEEN THE CIRCLE AND SQUARE MIGHT BE AT THE CORE OF THE EGYPTIAN ROYAL CUBIT

Carl Balowski | November, 2024

Abstract

The Approximated Circle-Square Template (ACST) is a mathematically and symbolically rich framework derived from ancient design principles, facilitating an elegant approximation of the relationship between circles and squares. Centered around "Nun," a primordial circle symbolizing the void, the ACST utilizes two squares—the inner "Remen Square" and the outer "Royal Cubit Square"—to achieve proportional relationships that approximate irrational constants such as π and √2 through rational whole fraction measures. By creating a 140-unit "Royal Cubit" rod, the ACST enables practical applications in design and construction, allowing designers to approach these constants with both accuracy and simplicity. The Royal Difference, calculated as the gap between the Royal Cubit and Remen squares, plays a key role in aligning the template with constants and ratios that mirror those found in ancient structures, like the Great Pyramid and Stonehenge.

This paper explores the ACST's proportional approximations for π, √2, √3, and √5, revealing how each iteration builds upon the preceding one through factors of √2, doubling every other iteration. In addition to π = 440/140, √2 = 140/99, √3 = 343/198, and √5 = 246/110, the ACST's pragmatic segmentation renders these measures highly accurate, approximating constants to within 99.98%. This approach shows how ancient builders may have used whole fraction "measures" as a form of math, with these ratios providing a precise, symbolic, and practically applicable model. The template's use of rational whole fractions brings to life a philosophy in which the cosmos (circle) and earth (square) harmoniously coexist through precision and balance. This construct underscores the ACST's utility as both a theoretical and applied "measuring rod," offering insight into how sacred mathematical constants might have guided the architectural and cosmological visions of ancient cultures.

Introduction

The Approximated Circle-Square Template (ACST) offers an intriguing mathematical and symbolic framework, drawing from ancient design principles to capture the relationship between a circle and a square. At the center of this template is a circle, referred to as "Nun" (blue), symbolizing the primordial and encompassing void or waters from which all else emerges. From this central, primordial and encompassing circle, two squares emerge: the inner square, called the "Remen Square" (red), and the larger, outer square called the "Royal Cubit Square" (green). By approximating irrational constants like π and √2 through rational ratios, the ACST reveals a method of bringing harmonious balance between the circle and square—a relationship revered by ancient architects and mathematicians. To illustrate the pragmatic use of geometric proportions within the ACST, I created a linear measurement rod, known as the Royal Cubit (Figure 1).

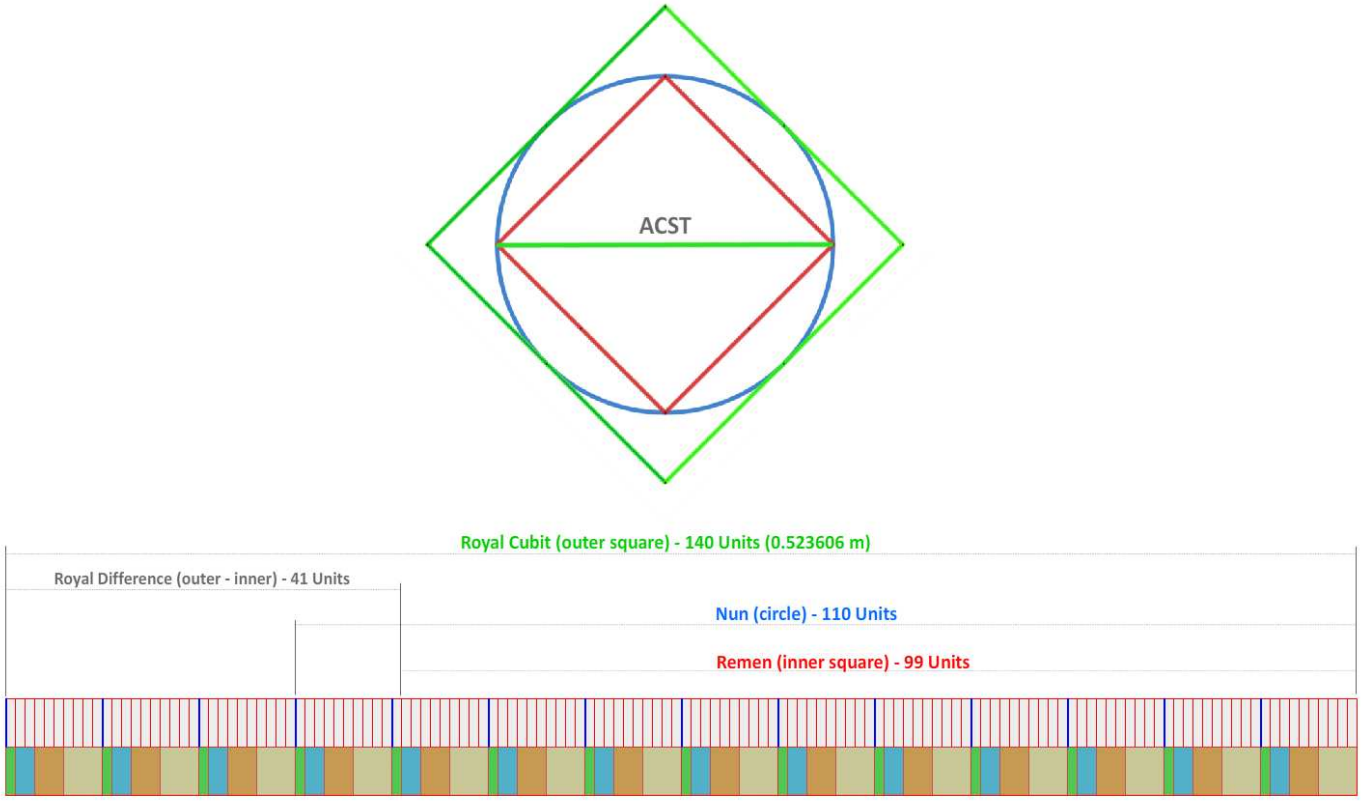


Figure 1

This 140-unit Royal Cubit rod serves as a practical tool for applying these proportions in design and construction. While the ultimate measurement of 0.523606 meters for the Royal Cubit rod is extremely precise, its divisions into irrational values, such as √2 (140/99) and π (110/35), are pragmatically

approximated to ensure usability in the physical world. Yet these divisions were not only practical but also meant to highlight the axiomatic and sacred relationship between the circle and square, symbolic of the heavens and earth. Divided into 140 units, the Royal Cubit rod translates the ACST's proportional relationships into a manageable linear system, enabling designers and builders to recreate these harmonies effectively.

Defining the Primary Components of the ACST

Nun, the primordial blue circle, has a diameter matching the side length of the Royal Cubit Square, creating an elegant proportional relationship between the circle and the squares. The Remen Square in red, with a side length of 99 units, is inherently positioned so that its diagonal is precisely equal to the side of the green Royal Cubit Square. The Royal Cubit Square, forming the outermost boundary, has a side length of 140 units, inherently matching the diagonal length of the Remen Square at 0.523606 meters. This side length is also the diameter of the circle and the diagonal of the Remen Square. At 0.523606 meters, it serves as a crucial measure, setting the standard by which all other elements in the ACST are approximated.

The Royal Difference—the difference in measure between the Royal Cubit Square and the Remen Square—is calculated as 140 - 99 = 41 units. This value is crucial, as it represents the proportional gap between the two squares, providing a foundational basis not only for the circle-square relationship but also for comparing and approximating the key mathematical constants involved.

Approximations in the ACST

To approximate √2, the ACST employs the fraction 140/99 = 1.41414, which is extremely close to the true value of √2, to within 99.99%. For π, it uses 440/140, or 22/7, which is within 99.95%, leading to a circumference of 440 units for Nun, given its diameter of 140 units. A quarter of this circumference—110 units— like how the side of square is ¼ its perimeter, is used in key comparisons, simplifying the mathematical relations in a way that was both symbolic and practical for ancient builders.

Proportional Ratios in the ACST

The relationships between the components of the ACST reveal ratios that align precisely with significant mathematical constants that we see in ancient construction, like the Great Pyramid, and the positioning of both Khufu and Khafre's pyramids as well as the placement and design of the Station Stones with Heel Stone at Stonehenge, the details which won't be listed here, but are available in my previous studies.

Ratio of the Royal Cubit (RC) Square to the Remen (Rm) Square:

The side length of the Royal Cubit Square (140 units) divided by the side length of the Remen Square (99 units) gives a ratio of 1.41414 : 1, which is a precise approximation of √2.

Ratio of the Remen (Rm) Square to the Royal Difference (RD):

The side length of the Remen Square (99 units) to the Royal Difference (41 units) gives a ratio of approximately 2.4146, precise to (1 + √2) : 1. This correlates precisely to the Silver ratio to within 99.98%.

Ratio of the Royal Cubit (RC) Square to the Royal Difference (RD):

The side length of the Royal Cubit Square (140 units) to the Royal Difference (41 units) produces a ratio of approximately 3.4146 : 1 which again approximates (2 + √2) : 1. This value is again notably precise to the Silver Ratio.

Ratio of the Royal Cubit (RC) Square to Nun (Nu):

The side length of the Royal Cubit Square (140 units) to the quarter circumference of Nun (110 units) yields a ratio of 1.272727 : 1, demonstrated as the height : half-base ratio of the Great Pyramid.

Ratio of Nun (Nu) to the Remen (Rm) Square:

The quarter circumference of Nun (110 units) divided by the side length of the Remen Square (99 units) yields a ratio of 1.1111 : 1, or inversely 0.9 : 1, correlating the half-diagonal to height of the Great Pyramid.

Ratio of Nun (Nu) to the Royal Difference (RD):

The Royal Difference (41 units) to the quarter circumference of Nun (110 units) yields approximately 2.682927, which aligns precisely with the value 0.9 : 2.4141, which is the lateral distance of the base of Khufu's pyramid to the northern face of Khafre's.

Ratio of Nun (Nu) to half the Royal Cubit (RC):

The measure of Nun compared to half the Royal Cubit is 1.5714 : 1, or π/2. This algins with the value of the Great Pyramid base to half height.

Symbolic and Practical Implications

Through these relationships, the ACST provides a powerful model for the ancient architects to approximate the otherwise irrational relationship between circles and squares, making it 'rational' in a practical sense. By segmenting the Royal Cubit into 140 units and approximating $\sqrt{2}$ and π with fractions, the ACST exemplifies an elegant solution to the circle-square problem. Furthermore, the color distinctions—Royal Cubit Square in green, Remen Square in red, and Nun in blue—help to visually clarify each component, enhancing the narrative and inviting symbolic interpretations.

In this way, the ACST can be seen as a mathematical and philosophical construct, embodying a worldview where the cosmos (symbolized by the circle) and the Earth (symbolized by the square) are brought into harmony through precise approximation and proportion. This template not only represents practical geometry but also reflects a worldview grounded in order, symmetry, and the belief that mathematics bridges the divine and the physical realms.

Iterative Doubling of the ACST and Approximations of $\sqrt{5}$ and $\sqrt{3}$

A unique aspect of the ACST design is that each successive iteration of the square increases by a factor of $\sqrt{2}$, and "every other" iteration doubles in linear measure. This iterative process can theoretically continue indefinitely (Figure 2), but even simple arithmetic with the first few iterations reveals remarkable approximations. Specifically, by applying the ACST, we find near-perfect approximations for $\sqrt{5}$ and $\sqrt{3}$.

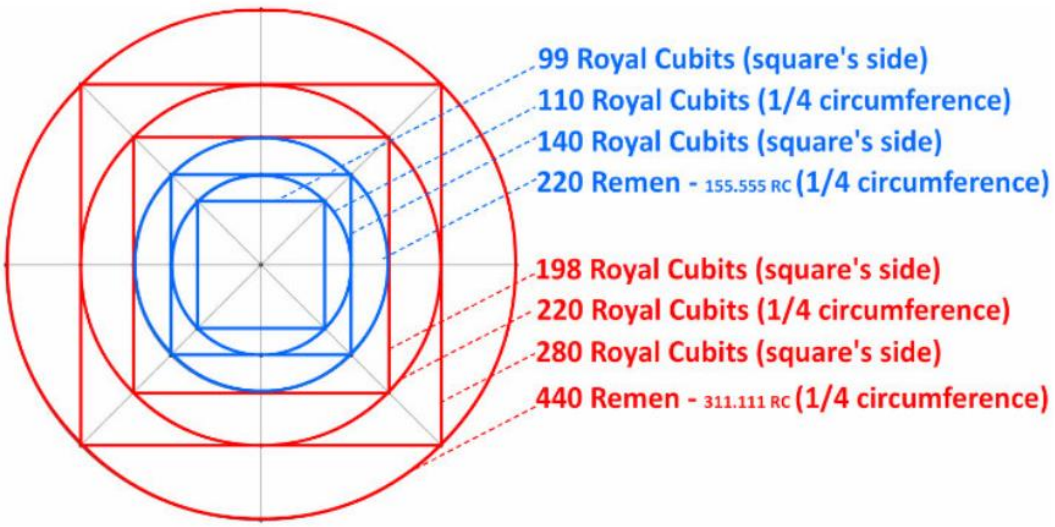


Figure 2

For √3, we start with 220 (the second iteration of Nun, or half the circumference from the first iteration) and add it to thrice the Royal Difference (41), giving 123, which together make 343. Dividing 343 by 198 (twice 99, the second iteration of 99) results in 1.73232, approximating √3 to within 99.98%, or 343/198.

For √5, taking six times the Royal Difference of the first iteration (or three times the difference of the second iteration), which equals 246, and dividing this by the first iteration of Nun (110), yields 246/110 or 2.23636, which approximates √5 to within 99.98%. Each of these approximations, √3 and √5, relies on an iteration of the Royal Difference, which defines the Silver Ratio, compared to an iteration of either Nun or the Royal Cubit.

In summary, the arithmetic to compute the ineffable irrational values using the *ACST Royal Cubit* is as follows:

$$\pi = 440 / 140 = (4\ Nu) / (1\ RC)$$

$$\sqrt{2} = 140/99 = (1\ RC) / (1\ Rm)$$

$$\sqrt{3} = 343/198 = (2\ Nu + 3\ RD) / (2\ RC)$$

$$\sqrt{5} = 246/110 = (6\ RD) / (1\ Nu)$$

Designing with Sacred Mathematical Constants

By using the ACST as a "measuring rod," we are able to design and build in alignment with all the "sacred" mathematical irrationals—including π, √2, √3, and √5—using rational whole fraction approximations, with simple arithmetic, that achieve a very high degree of accuracy. These approximations allow nearly perfect representations of the axiomatic relationship between the circle and square, symbolically connecting heaven and earth. The ACST thus serves as both a practical and symbolic tool, demonstrating how ancient builders may have approached complex mathematical relationships with an elegant, harmonious system grounded in rational approximations.

Conclusion

The Approximated Circle-Square Template (ACST) demonstrates that ancient builders may have employed a sophisticated yet practical mathematical framework to approximate irrational constants and to bridge the symbolic relationship between the circle (heaven) and the square (earth). Through the ACST, constants such as π, √2, √3, and √5 can be approximated with remarkable accuracy using whole fractions, transforming abstract irrational numbers into manageable "measures." This approach suggests that the ancient world's concept of "math" could include rational ratios used as physical measures, grounded in accessible arithmetic.

The Royal Cubit rod, segmented into 140 units, shows how the ACST principles could be applied in real-world design, as evidenced by the alignments in notable structures such as the Great Pyramid and Stonehenge. The Royal Difference, created by the gap between the Remen and Royal Cubit Squares, provides a foundational basis for the ACST's proportions, reinforcing its utility as a "measuring rod" to approximate complex mathematical relationships.

In both symbolic and practical terms, the ACST represents a worldview where order, harmony, and the axiomatic alignments between heaven and earth are achieved through rational approximation. The ACST invites further exploration into how ancient cultures approached arithmetic, geometry and the cosmos, revealing an elegant system that not only reflects mathematical precision but also embodies the philosophy and cosmology of the ancient world.