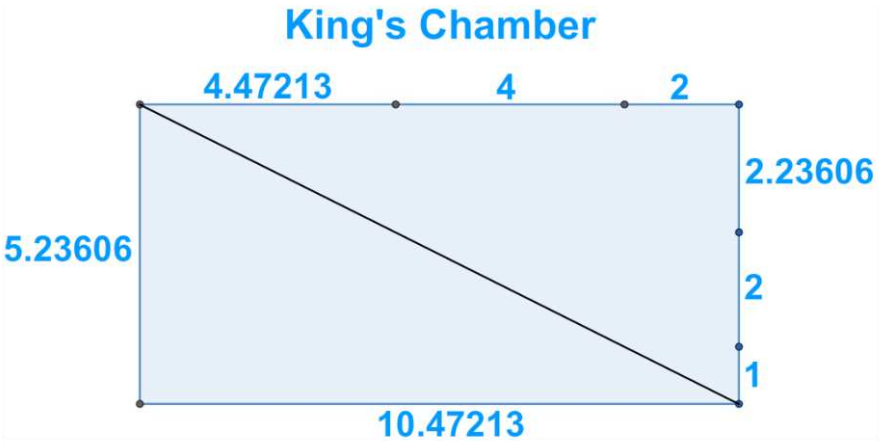
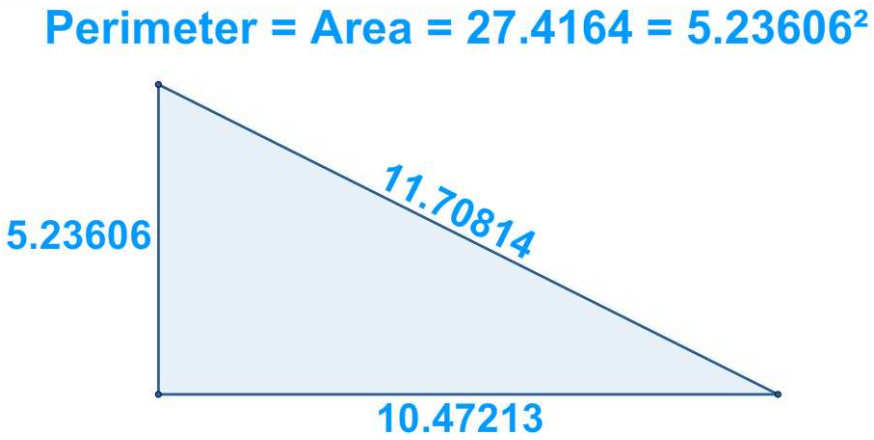






The Geometrical Trifecta

THE √5 RIGHT TRAINGLE, THE KING'S CHAMBER, AND THE POTENTIAL METRIC LEGACY OF THE ROYAL CUBIT

Carl Balowski | December, 2024

Abstract

This paper presents the discovery of a unique right triangle that satisfies a unique geometrical trifecta: its area and perimeter are identical, both equaling $(3+\sqrt{5})^2 = 27.4164$, while the square root of both values equals its shortest side, $3+\sqrt{5} = 5.23606$. This triangle originates from the foundational 1 : 2 : √5 triangle, whose perimeter also equals $3+\sqrt{5}$ (or 5.23606), and serves as the key to scaling a larger, proportionally identical triangle with remarkable properties.

The study reveals that this triangle's shortest side, 5.23606 meters, aligns precisely with 10 Royal Cubits, where 1 Royal Cubit equals 0.523606 meters. This measure is consistently found in ancient Egyptian metrology and aligns with the surveyed dimensions of the King's Chamber in the Great Pyramid of Giza. Documented meticulously by Flinders Petrie, the chamber's proportions reflect this triangle's geometry:

- The short side measures 5.23606 meters (10 Royal Cubits),
- The long side measures 10.47212 meters (20 Royal Cubits),
- The diagonal measures 11.70820 meters (√5 times the short side).

Furthermore, Glen Dash's survey of the exterior base of the Great Pyramid corroborates this measurement system, yielding an average Royal Cubit of 0.523606 meters when dividing the base sides by 440 cubits. Additional evidence comes from surviving artifacts, such as the Rods of Kha and Maya, which approximate this Royal Cubit value with remarkable precision.

The convergence of geometry, metrology, and architecture suggests that the King's Chamber may serve as a deliberate pointer to a primitive measure equivalent to 1 meter, conceptually encoded within its design. By exploring this extraordinary mathematical relationship and its correlation to ancient measurements, this study sheds light on the sophistication of ancient design principles, challenging assumptions about the knowledge and precision of the builders of the Great Pyramid.

Introduction

The interplay between geometry, mathematics, and ancient metrology has long fascinated scholars and historians. This study examines a unique right triangle that satisfies a rare mathematical trifecta: its area and perimeter are identical, and their square root equals the triangle's shortest side of $3+\sqrt{5}$ (or 5.23606). Remarkably, this configuration aligns with the proportions of the King's Chamber in the Great Pyramid of Giza, where the Royal Cubit, measuring 0.523606 meters, emerges as a fundamental unit.

By tracing the relationships between the King's Chamber triangle, the foundational $1 : 2 : \sqrt{5}$ triangle, and ancient Egyptian measurements documented by Flinders Petrie and Glen Dash, this paper explores the possibility that the King's Chamber encodes a primitive measure equivalent to 1 meter. Such findings invite further investigation into the advanced mathematical and architectural knowledge of ancient cultures.

Determining the Standard Royal Cubit

There is no general consensus on the exact length of the Egyptian Royal Cubit, nor is it clear whether its length varied across pharaohs, nomes, or dynasties. Nevertheless, it is widely accepted that most Egyptian Royal Cubits were approximately between 522 and 525 millimeters. To explore ancient metrology in the context of the Great Pyramid, it becomes necessary to adopt a specific "standard" length for the Royal Cubit used in the Great Pyramid's design. This standard is hypothesized based on a dataset of empirical evidence, representing what the original designers may have intended, and serves as a benchmark to compare with newly discovered evidence for refinement or adjustment.

This study adopts a value of 523.606 millimeters as the standard Royal Cubit for the Great Pyramid. This length is derived from a detailed comparison of Petrie's measurements of the King's Chamber and Dash's contemporary surveys of the pyramid's base, integrated with an analysis of relevant mathematical constants. This choice reflects a reasoned interpretation of historical and archaeological data, offering the most precise value for the Royal Cubit within this context.

Moreover, this general hypothesis is that the Great Pyramid functioned, whether intentionally or unintentionally, as a kind of "archive" for metrological standards, much like the later "archive de mètre" in France. This suggests that not only did later Egyptian Royal Cubits derive their lengths from this standard, but other measurement systems across the ancient world may also have originated as adaptations or modifications of the Royal Cubit used in the pyramid's construction. If true, this would position the Great Pyramid as a universal reference point, preserving a fundamental standard of

measurement that influenced metrological practices far beyond Egypt—an influence potentially reflected in the design of the Nippur Rod as will be examine in a forthcoming study.

To determine a 'standard' Royal Cubit used in the construction of the Great Pyramid, we rely on measurements from both the King's Chamber and the Great Pyramid's exterior base. Petrie's survey of the King's Chamber, converted to metric, provides ranges for the North and South walls between 10.4617 meters and 10.4846 meters. Dividing these measurements by 20 cubits yields a Royal Cubit length ranging from 523.085 mm to 524.230 mm, with an average of 523.657 mm. For the East and West walls, the reported range is 5.2316 meters to 5.2433 meters. Dividing these by 10 cubits gives an average Royal Cubit length of 523.745 mm. Combining all four walls, the overall average for the Royal Cubit in the King's Chamber is 523.701 mm, with a range between 523.085 mm and 523.745 mm.

Dash's 2015 survey of the Great Pyramid's exterior base provides mean side lengths ranging from 230.329 meters to 230.407 meters, resulting in an average base length of 230.368 meters.[1] Assuming the base was designed to be 440 cubits, this corresponds to a Royal Cubit length of 523.563 mm, and a range between 523.475 and 523.652 mm.

By averaging the King's Chamber dimensions (523.701 mm) with the exterior base dimensions (523.563 mm), a plausible 'average' Royal Cubit length of 523.632 mm emerges as a potential measure used in the design and construction of the Great Pyramid. For this study, however, 523.606 millimeters has been chosen as the ideal Royal Cubit (Figure 1). This value, being 99.995% of the average, was selected not only as a mathematical ideal derived from the survey data set but also because its proximity to the survey data allows for the consideration of both minuscule construction errors and seismic activity.

[1] Dash and Paulson, The 2015 Survey of the Base of the Great Pyramid. https://www.academia.edu/36742756/The_2015_Survey_of_the_Base_of_the_Great_Pyramid

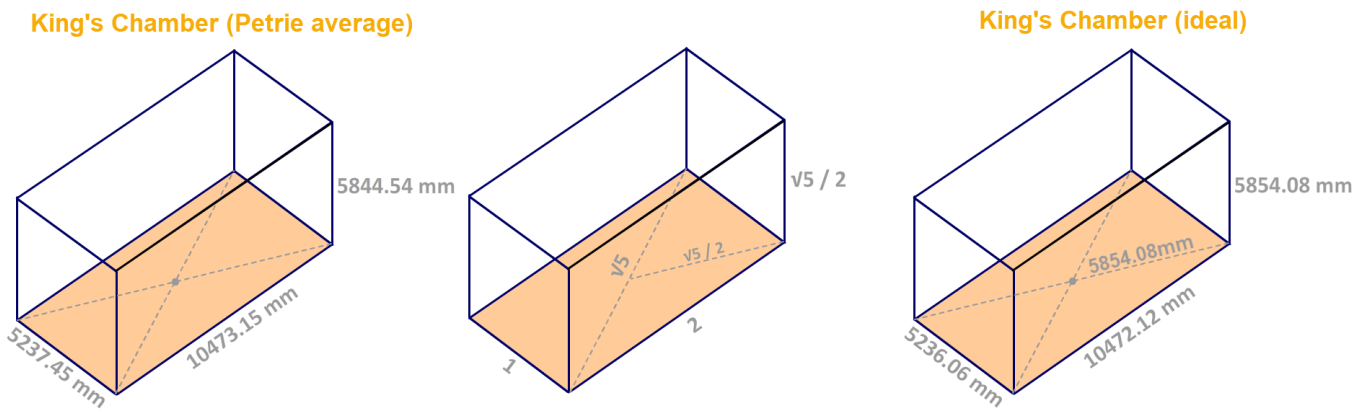


Figure 1

The selection is nearly perfectly centered within the survey ranges of both Petrie and Dash, with a minuscule difference of just 0.026 mm from the plausible average—a deviation smaller than half the width of a human hair and imperceptible to the naked eye. While this level of precision underscores the remarkable accuracy of ancient Egyptian metrology, it also accounts for the inherent reality that no construction, particularly one completed 4600 years ago, could achieve absolute perfection. This combination of practical and theoretical considerations provides a robust foundation for further investigation into the Great Pyramid's design and its broader influence on metrological systems across the ancient world.[2]

It is undeniable that the dimensions of the King's Chamber were deliberately designed according to the principles of the "double square," or a 1 : 2 : √5 triangle, where the floor forms a precise 1 : 2 rectangle. The diagonal of such a rectangle is defined by the √5, or approximately 2.23606, with half of this value being 1.11803. The ideal height of the King's Chamber appears to have been achieved by bisecting the two √5 diagonals of the rectangular floor and then extending each bisected diagonal vertically, resulting in a height at each of the four corners of exactly 1.11803 units relative to the shorter side of the rectangle, which is 1. This precise geometrical relationship underscores the advanced mathematical principles inherent in the chamber's design, with √5 serving as a cornerstone of its proportions and construction.

[2] Although not directly addressed in this study, using the measure of 523.606 mm for the Royal Cubit, along with Petrie's data indicating a mean angle of 51.844° for the casing stones on the north face of the Great Pyramid, yields a calculated height of 146.616 meters. Given Petrie's margin of error of ±0.0185° for the angle, the height could range based on angles between 51.825° and 51.862°. Notably, if the actual angle is 51.8427°—just 0.0017° less than Petrie's mean—this would correspond precisely to a height of 280 Royal Cubits (or 146.609 m). See
Petrie, W. M. F. (1884). The Pyramids and Temples of Gizeh. United Kingdom. P. 41-42

Given that the ideal short side of the King's Chamber is 10 Royal Cubits, or 5236.06 mm, this would make the ideal height exactly √125 Royal Cubits, or 11.1803 Royal Cubits, equating to 5854.08 mm. This calculated height aligns precisely within the range of Petrie's measurements for the chamber's actual height, further supporting the hypothesis of deliberate design rooted in the mathematical precision of 1 : 2 : √5.

The Rods of Kha and Maya

The rods of Kha and Maya (ca 1420 – 1330 BC) provide further evidence of the remarkable consistency in ancient Egyptian metrology, aligning closely with this study's chosen standard of 523.606 mm. Despite being produced more than 1,000 years after the construction of the Great Pyramid, their measurements show minimal deviations, likely reflecting the enduring influence of the Great Pyramid's Royal Cubit as a standard.

The rod from Maya's tomb, reported at 523 mm, deviates by only 0.606 mm below the standard, corresponding to a percentage error of -0.116% (Figure 2).



Figure 2

In contrast, Kha's gold-leafed rod measures 524 mm, exceeding the standard by only 0.394 mm, with a percentage error of +0.075%.[3] This slight overshoot could be attributed to the addition of the gold leaf, which likely served a symbolic and ceremonial purpose rather than purely utilitarian (Figure 3). These minor deviations—one slightly undershooting and the other slightly overshooting—suggest that both rods were crafted with a standard approximating 523.606 mm as the target, with almost invisible variations resulting from craftsmanship or materials.

[3] https://collezioni.museoegizio.it/en-GB/material/S_8647



Figure 3

Maya's rod, likely used for treasury oversight and construction during the New Kingdom, demonstrates remarkable precision, even though it was produced approximately a millennium after the construction of the Great Pyramid. Meanwhile, Kha's gold-leafed rod also carried ceremonial significance, underscoring the enduring reverence for the Royal Cubit as a symbol of authority and precision. The miniscule deviations from the standard highlight the central importance of the Royal Cubit in ancient Egyptian metrology, even as production methods and craftsmanship evolved over time. Additionally, the possibility of 'rounded' reported values in the literature may further underscores the exceptional accuracy achieved by ancient artisans.

The evidence from Petrie's surveys of the King's Chamber and Dash's measurements of the Great Pyramid's exterior base, combined with the analysis of the rods of Kha and Maya, strongly supports the conclusion that the Royal Cubit of 523.606 millimeters was the standard used in the Great Pyramid's design and construction. But why choose a measure for the Royal Cubit equivalent to 523.606 millimeters, or 0.523606 meters?

The recursive 1 : 2 : √5 triangle

Geometry reveals patterns of extraordinary beauty and symmetry that often hint at deeper relationships. In examining the 1 : 2 : √5 proportional triangle of the King's Chamber, a truly unique and novel mathematical trifecta emerges: its area and perimeter, both equal to 5.23606^2, are numerically identical, and their square root equals the triangle's shortest side (Figure 4). In other words, √Area = √Perimeter = Side.

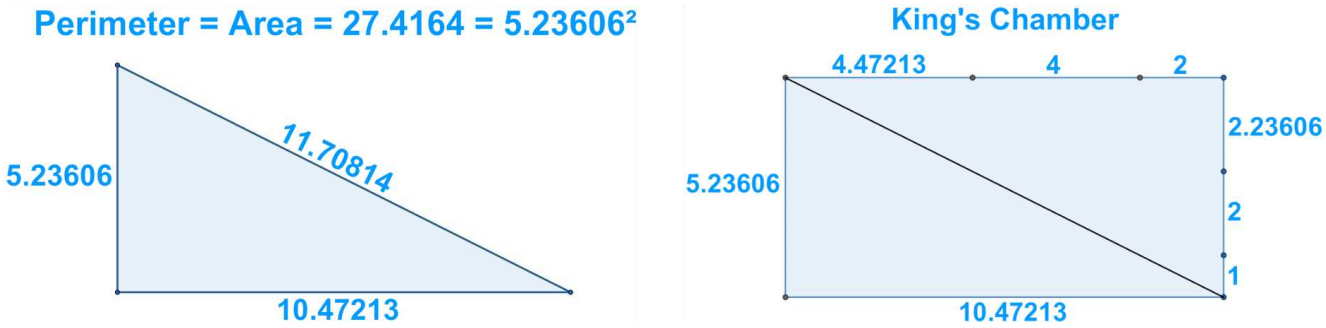


Figure 4

The triangle is defined as follows:

- Short side a = 3 + √5, which is 5.23606 meters,
- Long side b = 2a, which is 10.47212 meters,
- Hypotenuse c = √5a, which is 11.70820 meters.

Explicitly, the triangle satisfies three enigmatic interconnected properties:

1. The area equals $(3 + \sqrt{5})^2 = 27.4164$ m^2,
2. The perimeter also equals $(3 + \sqrt{5})^2 = 27.4164$ meters,
3. The square root of the area and perimeter equals the shortest side, $\sqrt{27.4164} = 3 + \sqrt{5} = 5.23606$ meters.

This triangle originates from the fundamental 1 : 2 : √5 triangle (Figure 5), whose perimeter majestically equals 3 + √5 (or 5.23606 meters).

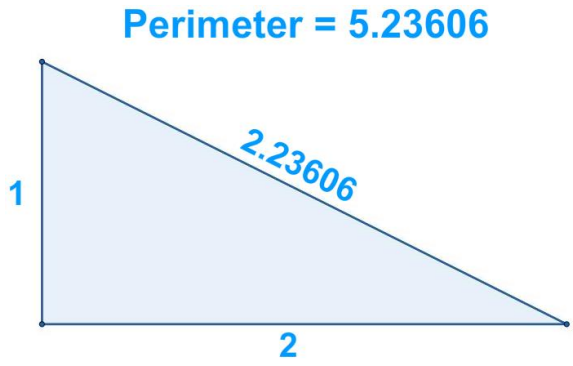


Figure 5

By using the total perimeter of this foundational 1 : 2 : √5 triangle to serve as the shortest side of a new, scaled triangle, a recursive relationship of remarkable simplicity emerges.

To explain this process step by step:

1. Begin with a measure of 1 meter to form the 1 : 2 : √5 right triangle in Figure 5, where the short side is 1, the long side is 2, and the hypotenuse equals √5 meters.
2. The total perimeter of this fundamental triangle is (1 + 2 + √5), or (3 + √5) which is equivalent to 5.23606 meters, or 10 Royal Cubits.
3. Using the perimeter of this fundamental triangle as the short side of a scaled triangle creates a new, larger triangle which precisely matches the dimensions of the King's chamber.

When measured in meters, the scaled triangle reveals a unique mathematical harmony:

- Its sides align precisely with the King's Chamber's dimensions:
    - The short side measures 5.23606 meters,
    - The long side measures 10.47212 meters (twice the short side),
    - The diagonal measures 11.70820 meters (√5 times the short side).
- Both the area and perimeter of this triangle are numerically identical, each precisely matching $(3 + \sqrt{5})^2 = 27.4164$ meters.
- As a result, the square root of both its perimeter and area equals its shortest side, $3 + \sqrt{5} = 5.23606$ meters.

The shortest side of the scaled triangle, 5.23606 meters, corresponds exactly to 10 Royal Cubits, where 1 Royal Cubit equals 0.523606 meters. This geometric and metrological relationship, rooted in the unique mathematical trifecta involving √5, suggests that the design of the King's Chamber may intentionally point to a foundational measure equivalent to 1 meter. Such remarkable precision raises the intriguing possibility that the builders had knowledge of a measure strikingly close to the modern meter and deliberately encoded it into the proportions of one of history's most significant architectural spaces.

To summarize, the triangle satisfies three interconnected relationships that form a unique mathematical trifecta:

1. **Area**:
   The area of the triangle is:
   $A = 1/2 \times a \times b = 1/2 \times (3 + \sqrt{5}) \times 2(3 + \sqrt{5})$.
   Simplifying:
   $A = (3 + \sqrt{5})^2 = 27.4164$.

2. **Perimeter**:
   The perimeter is the sum of all three sides:
   $P = a + b + c = (3 + \sqrt{5}) + 2(3 + \sqrt{5}) + \sqrt{5}(3 + \sqrt{5})$.
   Simplifying:
   $P = (3 + \sqrt{5})^2 = 27.4164$.

3. **Square Root of Area and Perimeter**:
   The square root of both the area and perimeter equals the triangle's shortest side:
   $\sqrt{A} = \sqrt{P} = \sqrt{27.4164} = 3 + \sqrt{5} = 5.23606$.

Thus, this triangle alone achieves a unique trifecta:

1. The area equals 27.4164,
2. The perimeter equals 27.4164,
3. The square root of both the area and perimeter equals the shortest side, approximately 5.23606.

Conclusion:

This discovery reveals a truly exceptional geometric relationship: a right triangle that satisfies a unique trifecta of mathematical harmony. Its area and perimeter are identical, both equaling 27.4164, while the square root of both values matches the triangle's shortest side, $3 + \sqrt{5}$, or 5.23606 meters. This elegant configuration connects theoretical geometry with ancient architecture and metrology in a way that demands careful consideration.

The significance of this discovery becomes even more profound when we observe its alignment with the dimensions of the King's Chamber in the Great Pyramid of Giza. The chamber, constructed with a 1 : 2 ratio, mirrors the proportions of this unique triangle. The surveyed measurements—meticulously documented by Flinders Petrie—confirm:

- The short side measures 5.23606 meters (10 Royal Cubits),
- The long side measures 10.47212 meters (20 Royal Cubits, twice the short side),
- The diagonal measures 11.70820 meters (√5 times the short side).

These precise dimensions reveal that the shortest side of the triangle, 3 + √5 = 5.23606 meters, directly corresponds to 10 Royal Cubits, with 1 Royal Cubit equaling 0.523606 meters. This value is no arbitrary coincidence. It emerges as a recurring unit in ancient Egyptian metrology and aligns remarkably with the measurements of key artifacts, such as the Rods of Kha and Maya, whose lengths closely approximate this standard.

Additionally, Glen Dash's survey of the exterior base of the Great Pyramid further reinforces this metrological consistency. Dash's measurements yield an average Royal Cubit of approximately 0.5236 meters when dividing the pyramid's surveyed base sides by 440 cubits. These findings, combined with Petrie's precision work, demonstrate a consistent system of measurement deeply rooted in the value of 5.23606 meters, or 10 Royal Cubits.

What makes this even more extraordinary is how the foundational 1 : 2 : √5 meter triangle—a simple geometric configuration with a perimeter of 3 + √5 meters—serves as the seed for this discovery. By scaling its perimeter to serve as the shortest side of a larger, proportionally identical triangle, we arrive at this mathematically and architecturally perfect configuration. The recursive elegance of this system reveals how geometry and metrology converge, suggesting a deliberate design principle at work.

The inescapable conclusion is that the dimensions of the King's Chamber match a measure remarkably equivalent to 1 meter, rooted in the mathematical beauty of 3 + √5. Whether intentional or not, the precision of these relationships invites us to reconsider the sophistication of ancient Egyptian design and their potential understanding of mathematical principles that mirror modern standards of measurement. This discovery not only deepens our appreciation for ancient metrology but also highlights the profound and universal nature of geometric harmony.