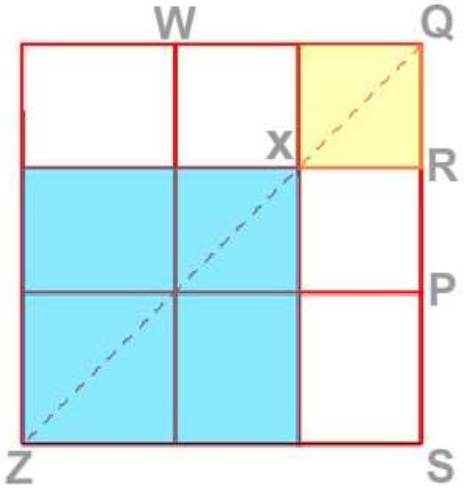




Rod Line	Length (mm)	Euclid 13:3
AG	847.21	4QS + 4QR
AC	261.8	QP + QS
CG	585.41	QP + 3QS
CE	309.01	5PS
EG	276.39	5QP - 2RS
FE	209.44	(RS + 3QR)/1.25
DB	120.7	QR + QX
ED	241.42	2QR + 2QX
EB	362.13	3QR + 3QX

The Nippur Rod and Egyptian Royal Cubit

BRIDGING ANCIENT METROLOGY, EUCLID AND THE GOLDEN RATIO

Carl Balowski | December, 2024

Abstract:

The Nippur Rod, an ancient copper measuring tool discovered in the Sumerian city of Nippur, provides compelling evidence of advanced metrological and geometric understanding in ancient Mesopotamia. Measuring 1103.5 mm in total length, its usable segment spans 847.21 mm, segmented into six parts that exhibit proportional relationships deeply rooted in mathematical and geometric principles, particularly the Golden Ratio and √5. This study explores the hypothesis that the rod's design was intentionally based on the Egyptian Royal Cubit scaled by the Golden Ratio, reflecting ceremonial rather than purely practical applications.

Analyzing its segmentations reveals remarkable correlations with Euclid's *Elements*, Book 13, and the proportional systems underlying ancient metrology. For example, key measurements of the rod align with divisions of √125 and √5, suggesting intentional design elements rooted in the Golden Ratio. Additionally, the segmentation of line CG (√5) into precise ratios mirrors relationships found in Thom's Megalithic Yard, potentially connecting the rod to broader ancient metrological systems.

The rod's design appears to honor sacred proportions, with its divisions corresponding to geometric relationships central to the construction of Platonic solids, as well as to the architectural ratios of the Great Pyramid. These findings support the hypothesis that the Nippur Rod was a symbolic artifact housed within a temple, serving as a standard for sacred offerings and demonstrating the interplay between arithmetic, geometry, and ancient metrological systems. By linking the Royal Cubit, the Golden Ratio, and Euclid's principles, the Nippur Rod offers a profound glimpse into the mathematical sophistication of ancient civilizations.

The Nippur Rod: A Glimpse into Ancient Metrology

The Nippur Rod, discovered in the ancient Sumerian city of Nippur, is one of the earliest known measuring tools, potentially showcasing an advanced understanding of geometry and metrology in ancient Mesopotamia.[1] Crafted from copper, the total rod measures 1103.5 mm in length and features a distinctive design: one end is roughly rounded into a curved protrusion resembling the pommel of a sword. This 'pommel' likely served to prevent slippage when the rod, which is notably heavy, was carried in one hand.[2] Adjacent to the pommel, before the first notch, is a space approximately 140 mm long, presumably designated as a grip area for ease of handling (Figure 1).



Figure 1

This design implies that precision measurements were intended to begin at the first notch after the 'grip' and end at the flat opposite end, resulting in a usable measuring length of 847.21 mm. Using this as a premise, the rod features seven demarcations, segmenting it into six 'ideal' parts with lengths (called lines), from right to left, as follows: AB = 208.693 mm, BC = 53.110 mm, CD = 67.595 mm, DE = 241.421 mm, EF = 209.440 mm, FG = 66.9500 mm (Figure 2).[3]

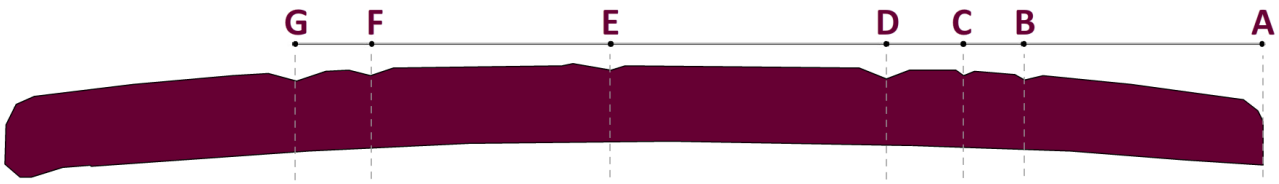


Figure 2

[1] All studies place this rod before 1600 BC, with some placing it as old as 2500 BC.
[2] The estimated weight of this solid copper rod is approximately 25 kilograms (55 pounds).
[3]The average deviation between the reported true value and the 'ideal segments' on this rod is just 0.12 mm—roughly the width of a human hair—with the largest deviation being only 0.3 mm from the reported 'true value.' The 'true value' lengths with skew from ideal lengths are as follows: AB = 209.00 mm (0.3 mm), BC = 53.00 mm (0.11 mm), CD = 67.50 cm (0.09 mm), DE = 241.50 mm (0.07 mm), EF = 209.50 mm (0.06 mm), FG = 67.00 mm (0.05 mm). See Zaide Duran, Umut Aydar, Digital modeling of world's first known length reference unit: The Nippur cubit rod, Journal of Cultural Heritage, Volume 13, Issue 3, 2012, Pages 352-356.

The six segments are clearly divided into two categories: "large" and "small." The small segments range between 53 mm and 68 mm, while the large segments range between 208 mm and 242 mm. The design of the rod is such that each large segment is paired with an adjacent small segment, creating three composite segments, each consisting of one large and one small segment (Figure 3).

Line AC (red) = 208.693 + 53.110 = 261.803 mm

Line CE (light blue) = 67.595 + 241.421 = 309.015 mm

Line EG (dark blue) = 209.44 + 66.95 = 276.390 mm

Line AG (total) = 8472.10 mm

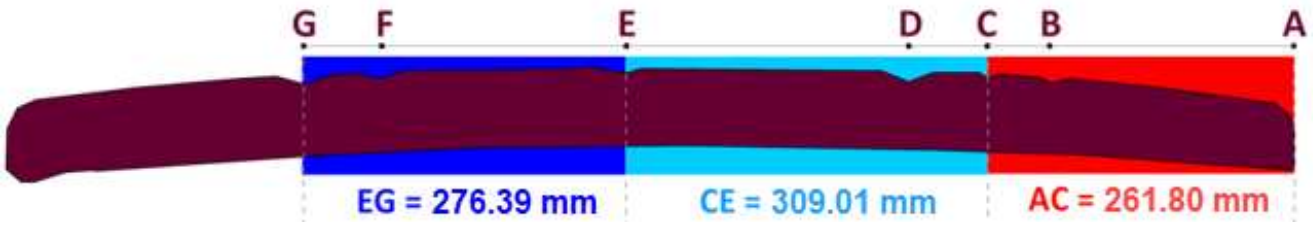


Figure 3

Evaluating the segmentation of the Nippur Rod provides a unique perspective, making it plausible that the rod, inspired by a √5 interpretation of the Egyptian Royal Cubit, functioned as a geometric tool honoring the Golden Ratio. Therefore, rather than serving as a practical instrument for larger scale metrology, the Nippur Rod might have held a ceremonial or symbolic role. Its material, design, precision, and heft suggest it was not meant to be portable, but rather stationed within the temple, possibly serving as a type of regional "archive de mètre." This rather massive, semi-precious rod, made from copper that was rare and highly valued in ancient Mesopotamia—could have been a standard against which votives or artisans compared their offerings or artifacts crafted for temple worship, such as altars, incense boxes, statues, and other sacred objects. The rod's notches provided a definitive measure, ensuring these items adhered to sacred proportions. Its composition, weight, and craftsmanship further imply its role as a revered object central to temple rituals and the maintenance of symbolic accuracy in offerings and sacred designs.

To understand the Nippur Rod's relationship to the Golden Ratio, it is essential to grasp a fundamental geometric concept rooted in fundamental base-10 arithmetic from antiquity: √125 - 5 = 6.1803. When

expressed as a proportion of 10, this value embodies the Golden Ratio, a pivotal mathematical and geometric relationship central to both ancient and Renaissance thought. This proportional relationship reflects the deep interplay between √125 (a multiple of √5) and the systematic divisions of 10, serving as a foundation for understanding the rod's symbolic and mathematical significance (Figure 4).

$$10 : (\sqrt{125} - 5) = \text{Golden Ratio}$$

$$(\sqrt{125} - 5) : (15 - \sqrt{125}) = \text{Golden Ratio}$$

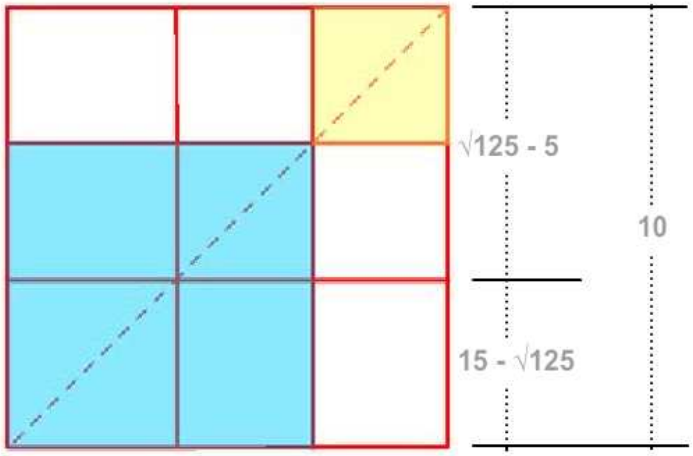


Figure 4

The expressions √125 - 5 and √5 - 1 are proportionally equivalent because they share a structural relationship: each involves subtracting a whole number from a square root, reflecting the same proportional relationship tied to the Golden Ratio. It is important to note that √125 is equal to 5 × √5, making it a scaled multiple of √5 by a factor of 5. As a result, the subtraction √125 - 5 directly corresponds to √5 - 1 when scaled.

Using √125 instead of √5 in arithmetic provides practical advantages, as it eliminates the need to work with fractions or values below 1, which are more challenging to compute without a place-value system. Scaling by a factor of 5 keeps the numbers larger and more compatible with the base-10 reckoning and arithmetic favored by the ancient Egyptians. This approach simplifies proportional relationships and manual calculations, aligning naturally with their numerical and metrological systems.

Manifestations of the Divine Proportion

The golden ratio, ϕ, (approximately 1.618033), is a mathematical constant that resonates throughout nature, geometry, and the arts, symbolizing harmony, balance, and efficiency. Its presence is not only derived from the √5 as demonstrated above but also from the Fibonacci sequence, where each term is the sum of the two preceding ones (1, 1, 2, 3, 5, 8, 13, …), underscores its foundational role in natural patterns. As the sequence progresses, the ratio of consecutive terms 3:5, 5:8, 8:13, etc… converge to the Golden Ratio (or ϕ\phi), forming the basis for many of its manifestations.

In geometry, the significance of ϕ (phi) is deeply rooted in Euclid's *Elements*. This foundational mathematical work culminates in the construction of the dodecahedron, or the quintessence, a Platonic solid whose proportions are governed by the golden ratio. The dodecahedron is not merely a geometric curiosity but a testament to the importance of ϕ in ancient mathematical thought. By building towards this 'divine' structure, the *Elements* make the golden ratio it's implicit goal, reflecting its role as a cornerstone of proportionality and harmony. This culmination occurs in Book XIII, where the golden ratio emerges as the critical element for constructing perfect geometric forms.

In the natural world, ϕ manifests with remarkable precision. The DNA molecule, the blueprint of life, measures 34 angstroms in length and 21 angstroms in width per full cycle of its double helix— 34 : 21, consecutive Fibonacci numbers whose ratio approximates ϕ. This proportionality exemplifies the efficiency and harmony inherent in life's design. Plants, too, reflect the divine proportion in their growth and structure: flowers often have Fibonacci numbers of petals—like daisies with 34 or buttercups with 5—and sunflower seed arrangements adhere to logarithmic spirals based on ϕ, optimizing space and light exposure. The 'golden angle' of 137.5°, derived from ϕ, governs these spirals, demonstrating nature's reliance on this proportion for growth and functionality.

Beyond Earth, ϕ extends into the cosmos. Spiral galaxies exhibit arms that follow logarithmic spirals, with successive radii often approximating ϕ, demonstrating gravitational efficiency. Even the earth's relation with the Moon and the planet Venus has semblance with the divine proportion. Similarly, many sea creatures and nautilus shells grow in logarithmic spirals adhering to ϕ, reinforcing its universality. Even human anatomy reflects the golden ratio: the ratio of total height to the height from the navel to the feet approximates ϕ, as do many facial proportions, influencing perceptions of beauty and symmetry.

Unity and Separation as Being.

The concept of the "whole" is intrinsically tied to the golden ratio, representing an archetypal unity—complete and indivisible. Before division, the whole exists as an ineffable state, devoid of comparison or external relationships. To express itself, the whole must divide, introducing a "cut" that creates otherness. This division, unique in the case of the golden ratio, ensures that the relationship between the whole and its parts preserves the essence of unity: the whole divided by the larger part equals the larger part divided by the smaller. This self-referential property ties the divided parts back to the original whole, maintaining harmony and unity even in separation.

This recursive property allows the larger segment to be divided further in the same proportion, creating infinite chains of proportionality. This fractal-like structure mirrors the self-replicating patterns in nature as stated above. Philosophically, the golden ratio embodies a paradox: the whole self-sacrifices its unity to create the "other," yet the parts remain inseparably tied to the whole.

The Friar's Quest

The 15th-century Franciscan friar and mathematician Luca Pacioli, like Leonardo of Pisa (Fibonacci) before him, in his book entitled the Divine Proportion, which was illustrated by Leonardo da Vinci, explored the Golden Ratio and its relation to $\sqrt{5} : 2$ (or $\sqrt{125} : 10$) in depth, providing an arithmetical framework to complement Euclid's ancient geometric approach to the Golden Ratio and its role in constructing the Platonic solids in *Elements*, Book XIII. Pacioli's work can be seen as a re-presentation of the base-10 arithmetic likely used by Euclid, and perhaps even the Ancient Egyptians, when formulating their geometric constructions. While the *Elements* itself does not include arithmetic—being based purely on deduction and geometric reasoning—Pacioli offers the numerical insights which likely underpinned Euclid's logic.

Early in his demonstration, as shown in Figure 4, Pacioli highlights the division of 10 into two segments:

Whole = 10

Larger = $\sqrt{125} - 5$ (or 6.1803)

Smaller = $15 - \sqrt{125}$ (or 3.8197).

These segments conform to the Golden Ratio, where the larger segment relates to the smaller in the same proportion as the whole (10) relates to the larger, illustrating the profound connection between arithmetic and the geometric principles central to the *Elements*.

This proportional relationship of the Golden Ratio as 10 : (√125 – 5) extends even further. For example, the ratio of (√125 + 15) : (√125 + 5) also reflects the Golden Ratio. Similarly, the ratio of (√125 + 15) : 10 corresponds to the square of the Golden Ratio, a relationship geometrically demonstrated in *Elements*, Book XIII, Propositions 3, 4 and 5. Moreover, the ratio of 42.3606 : 26.1803, which also embodies the Golden Ratio, arises from the relationship between (2√125 + 20) : (√125 + 15). These divisions and proportions are not arbitrary; they are deeply rooted in arithmetic and geometric principles, forming the foundation for defining and constructing the Platonic solids.

Arithmetic Meets Geometry: Constructing Proportions

Pacioli expands on and intertwines these divisions to demonstrate how the Golden Ratio, which he calls the Divine Proportion, permeates much of Euclid's Book 13. In one example, inspired by *Elements*, Book XIII, Proposition 3, he presents an elegant construction illustrating how the area of two squares—one five times larger than the other—can be integrated within the area of a single larger square.

Breaking Pacioli's analysis into units of 5 (Figure 5), the smaller yellow square, with a side length of 5 mm (line QR), has an area five times smaller than the larger blue square, which has a side length of √125 mm (line RS). Remarkably, both squares are nested within a larger square with a side length (line QS) of √125 + 5 mm, measuring 16.1803 mm. Line QS is segmented according to the Golden Ratio at P, where the proportion QS : QP, or 16.1803 : 10, is the same as QP : PS, or 10 : 6.1803. The ratio of the blue square to the yellow square, being 5 : 1, is given the breath of life by Euclid through bisecting the larger section of the Golden Ratio (line QP) into two equal parts at R, so that QR equals RP, and the ratio of QR : RS is 1 : √5 (or 1 : 2.23606), making the total line QS equivalent to 3.23606 (or 1+√5), or 2 x 1.61803.

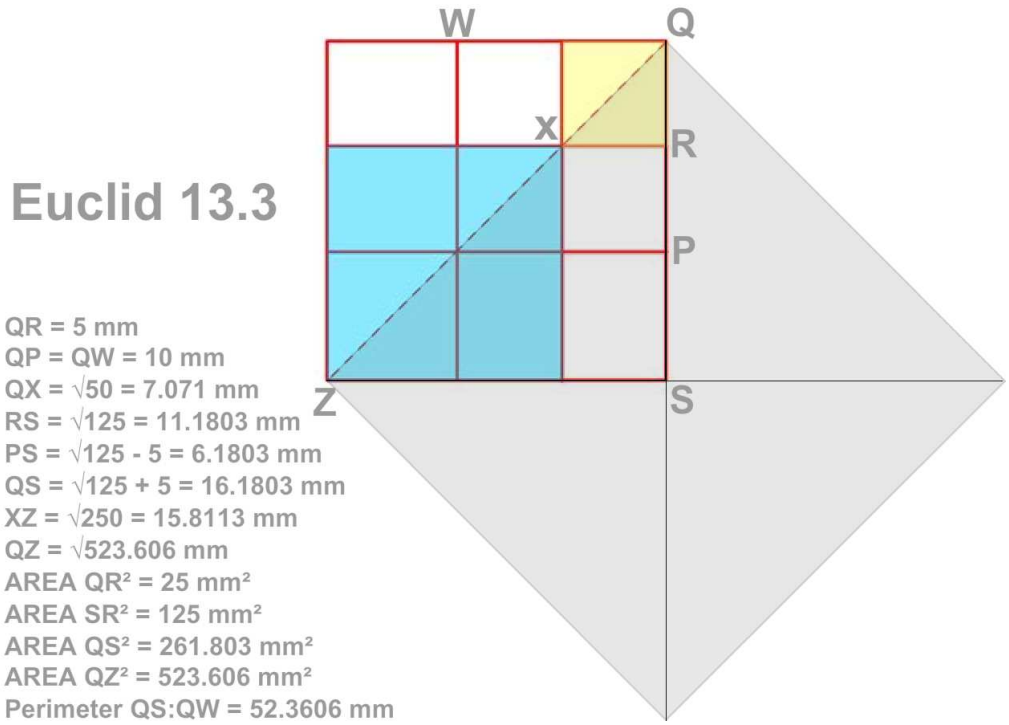


Figure 5

So, for the sake of this study, let us presume that the larger section of the Golden Ratio, line QP, is equivalent to 10 mm.[4]

- Line QP is 10 mm.
- Half of QP, which forms the side length of the smaller yellow square (line QR), is naturally 5 mm.
- The larger blue square, with a side length √125 mm, has an area of 125 mm², which is five times greater than the smaller yellow square with an area 25 mm².

As can be seen in the above illustration, the Euclid's geometric construction considering Pacioli's arithmetic highlights the deep connection between the Golden Ratio and √125, reinforcing their interplay visually and proportionally. Students familiar with the standard formulation of the Golden Ratio as (√5 + 1) : 2, will recognize its equivalent expression here as (√125 + 5) : 10, or QS : QP.

[4] 10 mm is, of course, equivalent to 1 centimeter. Since the square of QP (or 10²) has an area of 100 mm², we can describe the area of the square of QP as 1 square decimeter, even though the term "decimeter" is not commonly used today.

However, one intriguing detail that Pacioli of course was not aware of, but becomes evident upon closer examination, is the uncanny mathematical relationship of this construction to the presumed Royal Cubit of 523.606 millimeters as articulated in my earlier studies. The total square on the line of QS has an area of 261.803 mm², meaning that the square on the diagonal QZ—which is naturally double the area of QS—has an area of 523.606 mm², which is precisely the length of the Royal Cubit in millimeters. Furthermore, the rectangle formed by QS : QW, being a Golden Ratio rectangle, has a perimeter of 52.3606 mm, which is exactly one-tenth of the Royal Cubit.

Connection to the Platonic Solids

These geometric proportions laid out by Euclid in Book 13 and later articulated by Pacioli are rooted in the Divine Proportion, and are essential for constructing the Platonic solids, particularly the dodecahedron and icosahedron. The interplay between √125 and the number 5 in defining the Golden Ratio governs the relationships between edges, faces, and angles of these solids. Euclid uses these proportions early in *Elements*, Book XIII propositions 1 through 5, to demonstrate how the Golden Ratio enables the precise divisions necessary to form the solids' unique geometries. Pacioli builds on these constructions by assigning numerical values, showing how these geometric principles can be "visualized" arithmetically.

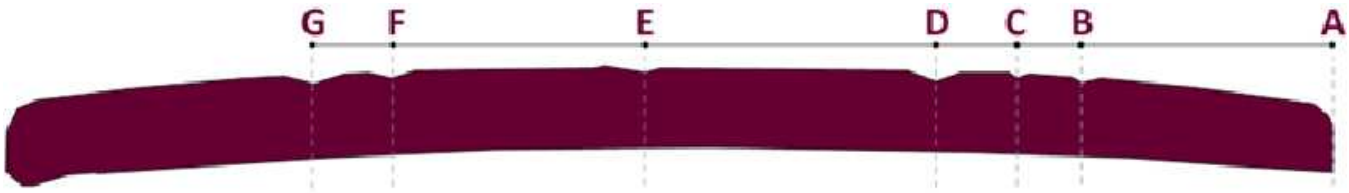
Euclid, whose goal in the *Elements* was ultimately to build a dodecahedron using the Divine Proportion, avoided explicit numeric arithmetic in the *Elements* to maintain a focus on pure deductive geometric reasoning. However, it is probable that the same arithmetic which guided both Leonardo de Pisa and Pacioli's analysis also played a background role in Euclid's work to test and prove geometric principles.

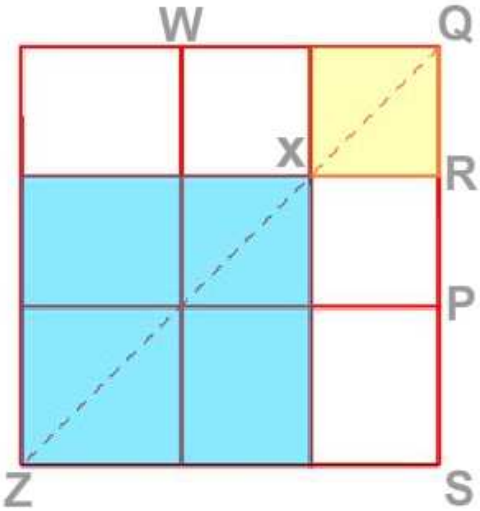
With this foundational understanding of how the Divine Proportion relates to √5, we can now examine the Nippur Rod with the hypothesis that its core design may be rooted in the application of the Egyptian Royal Cubit, influenced by √5 segmentations reminiscent of those described in Euclid's Book 13. To explore this connection further, let us revisit Euclid 13.3, viewing it through the lens of the various segmentations of the Nippur Rod, to see if we can extrapolate values that link the two together.

The Nippur Rod, the Royal Cubit and Euclid

In Figure 6 the first thing to note is that the measurable section of the Nippur Rod, line AG, is 847.21 millimeters, which corresponds exactly to 523.606 mm × 1.61803 (the Royal Cubit multiplied by the Golden Ratio). This strongly suggests that it is reasonable to begin our investigation with the presumption that the measuring section of the Nippur Rod was intentionally designed as the Egyptian Royal Cubit scaled by the Golden Ratio.

This observation alone is striking, though it might initially be dismissed as coincidence. However, when we note that AG is also segmented at point C into a √125 : 5 (or √5 : 1) ratio, with line AC measuring 261.80 millimeters, the significance becomes clearer. This length, 26.180 mm, matches the length of line AC on the Nippur Rod and corresponds to exactly half of an Egyptian Royal Cubit. Moreover, the entire Nippur Rod is segmented in a √125 : 5 ratio, or CG : AC, mirroring this proportional relationship.





Rod Line	Length (mm)	Euclid 13:3
AG	847.21	4QS + 4QR
AC	261.8	QP + QS
CG	585.41	QP + 3QS
CE	309.01	5PS
EG	276.39	5QP - 2RS
FE	209.44	(RS + 3QR)/1.25
DB	120.7	QR + QX
ED	241.42	2QR + 2QX
EB	362.13	3QR + 3QX

Figure 6

As we will see, dismissing this segmentation as mere coincidence becomes increasingly challenging when all the other segmentations are analyzed through the framework of Euclid Book 13, particularly Euclid 13.3. As shown in Figure 5, this proposition demonstrates that the geometric construction of a quintuple

square (in blue) depends on dividing the larger segment of the Golden Ratio (the Royal Cubit) in half, creating the yellow square. As previously mentioned, this bisection ensures that the entire line, when compared to half of the larger segment, forms a $(\sqrt{5} + 1) : 1$ ratio, or approximately 3.236 : 1.

This relationship is reflected in the Nippur Rod (Figure 7): if line AC is considered half a Royal Cubit (because it measures 261.8 mm) and assigned a value of 1, then line AG, representing a full Royal Cubit multiplied by 1.61803, results in 3.236, with line CG equating to 2.236 (or $\sqrt{5}$). Alternatively, if AC is assigned a value of 5, the full rod would total 16.1803 (or $\sqrt{125} + 5$), and CG would be 11.1803 (or $\sqrt{125}$). Regardless of the assignment, the same proportional relationship emerges, strongly suggesting that the rod's design might be rooted in deliberate geometric principles based on the Royal Cubit's connection to $\sqrt{5}$ and the Divine Proportion.

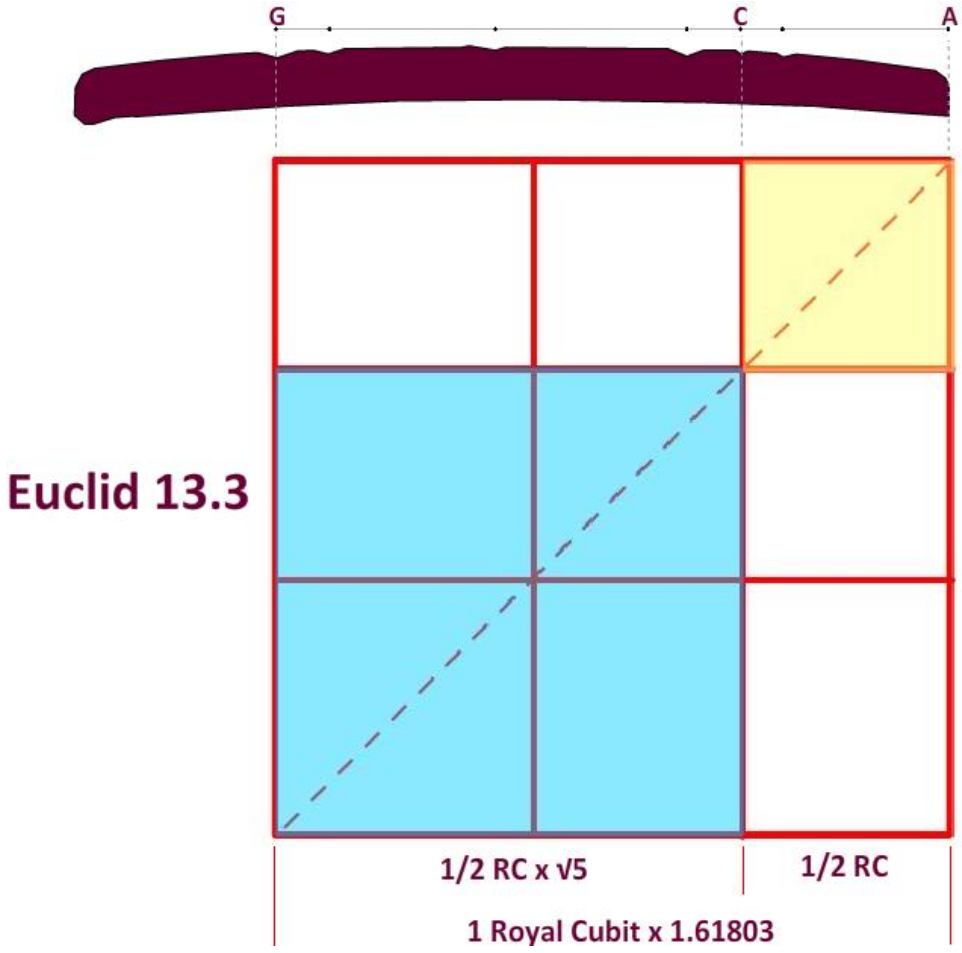


Figure 7

Considering other correlations with the segmentation of the rod (Figure 8 – 11), we observe that line CG, which represents 2.236 or √5, is nearly split in half, forming segments CE and EG, measuring 309.01 mm and 276.39 mm, respectively. You may already notice that the measure of CE at 309.01 mm is equivalent to 5PS from Euclid 13.3, which is half of 61.803, or (√125 – 5)/2. This is remarkable on its own, but it becomes even more compelling when we note that CE is precisely 1.11803 times longer than line EG (276.39 mm).

Even more intriguing, 276.39 mm corresponds exactly to 1/3rd of Thom's proposed value for the Megalithic Yard, calculated at 829.18 mm (2.7204 feet). This suggests that line EG represents 1/3rd of a Megalithic Yard, a correlation that seems far too precise to dismiss as coincidence. Much like the wall-to-height ratio of the King's Chamber, this implies that line CG, representing √5, is further divided into a 1 : 1.11803 ratio, where the value of 1 corresponds to 1/3rd of a Megalithic Yard.

Could this 1/3rd of a Megalithic Yard represent a 'Megalithic Foot'? If so, it would imply that the larger section of the rod (line AC) on the right end corresponds to ½ of a Royal Cubit, while the larger section on the left end (line GE) corresponds to 1/3rd of a Megalithic Yard. This fascinating connection unites these two ancient measures through the geometric principles of Euclid 13.3, suggesting that the creators of the rod may have been aware of both measures and their relationship.

As a final observation, Euclid 13.3 also includes a diagonal that traverses the diagram, illustrating the proportional relationships between the squares. This diagonal, segmented at point x, aligns the measurements of the middle section of the rod precisely with the proportions **75 : 50 : 25 : 14 : 11**. These proportions not only reflect the height-to-base ratio of the Great Pyramid but also reveal the Silver Ratio when line ED (2QR + 2QX) is compared to QP.

I leave it to you, dear reader, to reflect on these extraordinary mathematical correlations engraved into this remarkable piece of ancient copper.

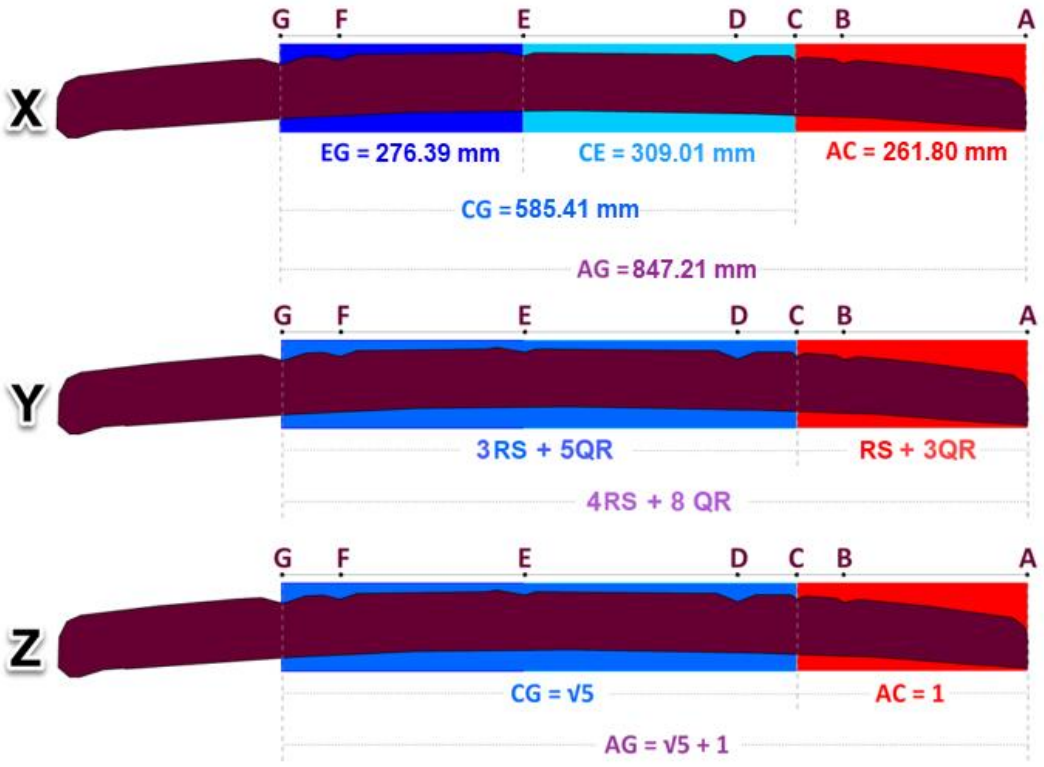


Figure 8

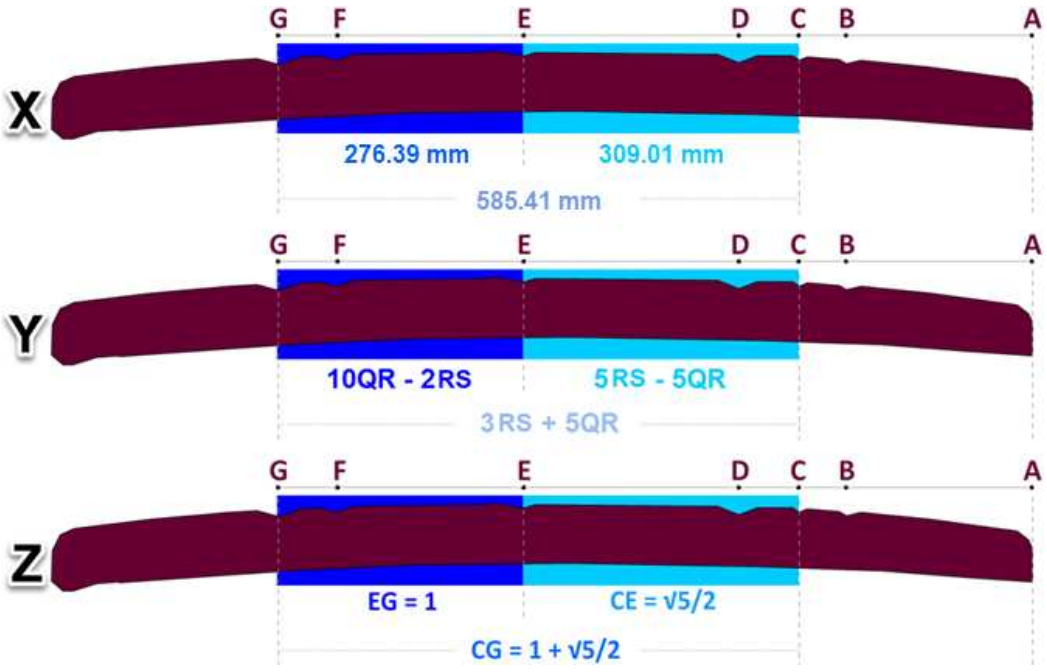


Figure 9

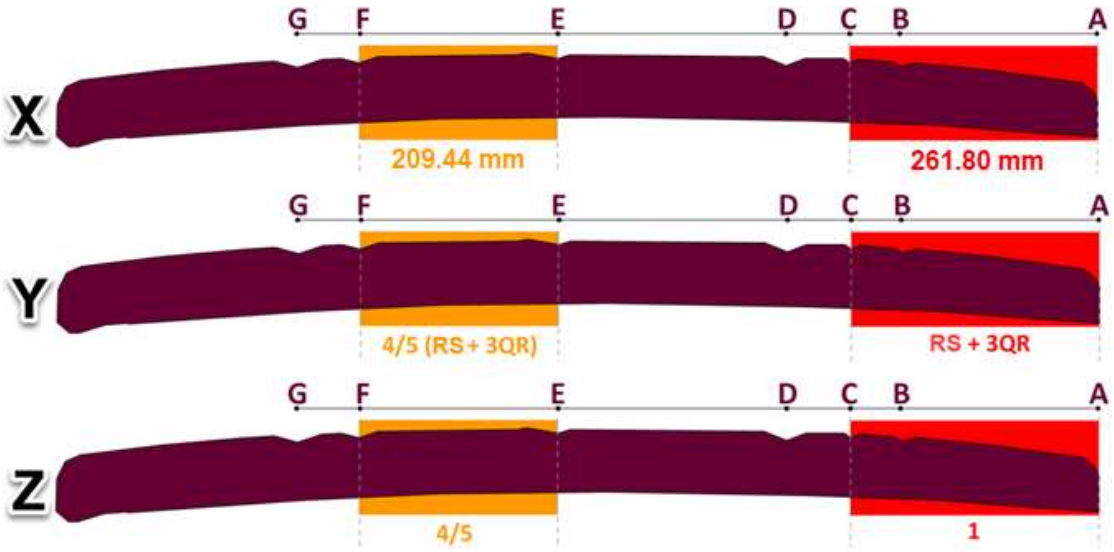


Figure 10

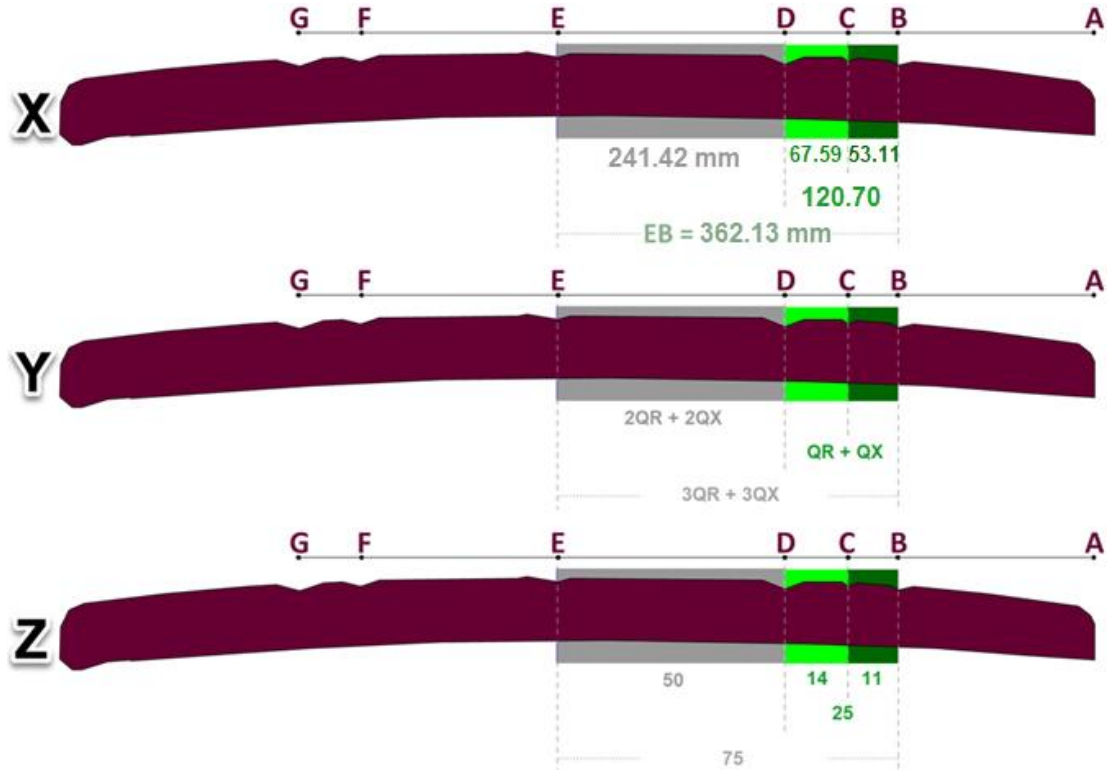


Figure 11